Unit 11 Probability Review

Name ______________________________ Per __

The **ADDITION RULE**:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

The MULTIPLICATION RULE:

$P(A \cap B) = P(A) \cdot P(B)$

$P(A \cap B) = P(A) \cdot P(B|A)$

CONDITIONAL RULE:

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Use the two way table showing the relationship between tattoos and hepatitis C in the state of Texas to answer questions # 1-2.

	Has hepatitis C	no hepatitis C	
Got tattoo at parlor	17	35	**52**
Got tattoo elsewhere	8	53	**61**
Has no tattoos	22	491	**513**
	47	**579**	**626**

1. What is the conditional probability of having hepatitis C given no tattoos? P(H|NT)

a. $\cancel{\dfrac{22}{626}}$ b. $\dfrac{22}{513}$ c. $\cancel{\dfrac{47}{626}}$ d. $\dfrac{47}{513}$

grand

2. What is the joint probability of having no hepatitis C and having no tattoos?

a. $\dfrac{491}{626}$ b. $\cancel{\dfrac{491}{513}}$ c. $\dfrac{579}{626}$ d. $\dfrac{513}{626}$

3. A survey of 57 sixth graders was done to determine which subject was their favorite. The results are shown in the table below sorted by gender.

	Math	English	Social Studies	Science	Total
Female	8	6	10	6	30
Male	10	4	9	4	27
Total	18	10	19	9	57

Using the table, show probability rule calculations for each of the following. Also identify the type of probability being described.

Calculation — Type

a. P (female) $30/57 = \boxed{.526}$ marg

b. P (female $\overset{\text{and}}{\cap}$ math) $8/57 = \boxed{.140}$ joint

c. P (male OR English)
$P(m) + P(E) - P(m \,\&\, E) = \dfrac{27}{57} + \dfrac{10}{57} - \dfrac{4}{57} = \boxed{.579}$ addition

d. P (science | male) $\dfrac{P(S \cap m)}{P(m)} = \dfrac{4}{27} = \boxed{.148}$ conditional

e. P $\underbrace{\text{(NOT social studies)}}_{\text{all others}}$ $\dfrac{18 + 10 + 10}{57} = \boxed{.667}$ marg

A person pulls a randomly selected marble out of a bag that has 5 blue and 7 red marbles, and does not replace it. A second person then randomly selects a marble from the bag.

(a) Use rules to calculate the probability of both people having pulled red marbles.

Conditional or independent? Cond.

$P(R \text{ and } R)$

$P(R) \cdot P(R|R)$

$\left(\frac{7}{12}\right)\left(\frac{6}{11}\right) = .318$

(b) Both marbles are returned to the bag, and the process is repeated, but this time the first person returns her marble to the bag before the second person selects. Use rules to find the probability of selecting 2 red marbles under this scenario.

Conditional or independent? Ind

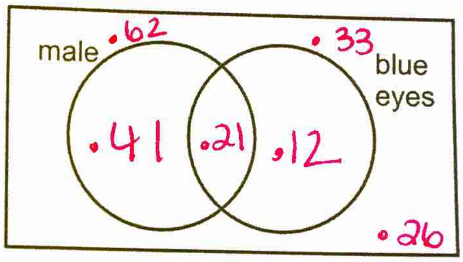
$P(R \text{ and } R)$

$P(R) \cdot P(R)$

$\left(\frac{7}{12}\right)\left(\frac{7}{12}\right) = .340$

5. The probability of randomly choosing a male marching band member is 0.62. The probability of randomly choosing a marching band member who has blue eyes is 0.33. The probability of finding a marching band member who is neither male nor has blue eyes is .26. Provide each answer to ***hundredths***.

a) Complete the Venn diagram to represent this situation.



b) What is the probability of finding a randomly chosen marching band member is a male who also has blue eyes?

.21

c) What is the probability of finding a randomly chosen marching band member is a male who does NOT have blue eyes?

.41

$1 - .26 = .74$

$.62 + .33 = .95$

$-.74$

$.21$ overlap

d) Is being a male independent from having blue eyes?.

$P(m)$

$.62$

not ind $\neq$

$P(m|B) = \frac{P(m \text{ and } B)}{P(B)}$

$= \frac{.21}{.33} = .64$

6. Given events A and B, such that $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \cup B) = 0.8$, determine whether A and B are independent or dependent.

$P(A) \cdot P(B) = P(A \cap B)$

$(.6)(.5) = .3$

$.3 = .3$

Indep

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$.8 = .6 + .5 - x$

$x = .3 \rightarrow P(A \cap B)$

7. The results of a survey of the student body at Central High School about television viewing preferences are shown below.

	Comedy Series	Drama Series	Reality Series	Total
Males	95	65	70	230
Females	80	70	110	260
Total	175	135	180	490

$P(M) = \frac{230}{490} = .469$

$P(M|R) = \frac{P(M \& R)}{P(R)} = \frac{70}{180}$

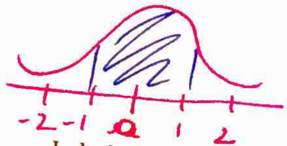
$= .389$

Are the events "student is a male" and "student prefers reality series" independent of each other? Justify your answer. no, $\neq$

Normal Distribution Curve:

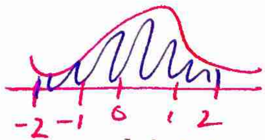
About 68 % of the area is between z = -1 and z = 1
("within" 1 StdDev of the mean)

Labeled, shaded sketch:



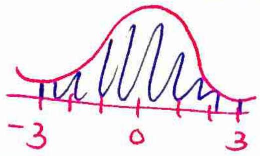
About 95% of the area is between z = -2 and z = 2
("within" 2 StdDevs of the mean)

Labeled, shaded sketch:



About 99 % of the area is between z = -3 and z = 3
("within" 3 StdDevs of the mean)

Labeled, shaded sketch:



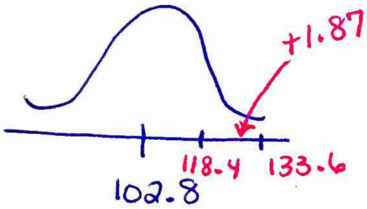
8. Which of the following calculator functions could be used to find the percentile rank for a person who has an IQ score of 112? IQ scores are normally distributed with a mean of 100 and a standard deviation of 15.

Shade below

a. invNormal (112, 100, 15)
b. normalcdf (100, 112, 100, 15)
c. normaldcf (-1E99, 100, 112, 15)
d. normalcdf (-1E99, 112, 100, 15)

X 9. A population has a mean of 102.8 and a standard deviation of 15.4. If a data point has a z-value of 1.87, what is the non-standardized value of the data point?

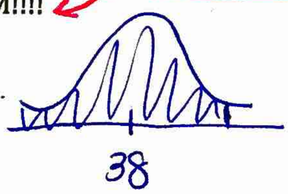
(1) ~~28.8~~ (2) 131.6 (3) ~~86.7~~ (4) 152.3



102.8 + 15.4
= 118.4
+ 15.4
133.6

10. The weights of four year old boys are normally distributed with a mean of 38 pounds and a standard deviation of 4 pounds. What weight represents the 90th percentile of a four year old? Show a shaded sketch and calculations/calculator inputs. **INVNORM!!!!**

given the percentile



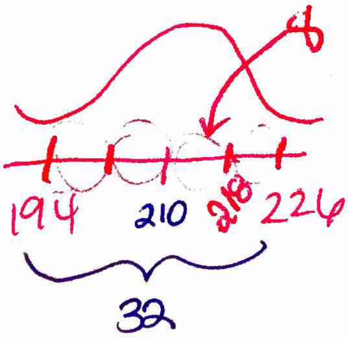
InvNorm (.9, 38, 4)

43 lbs

11. The lengths of songs on the radio are normally distributed. Within this curve, 95% of the songs have lengths between 194 and 226 seconds. Find the mean song length and the standard deviation.

2 z-scores out

$226 - 194 = \frac{32}{2} = 16$



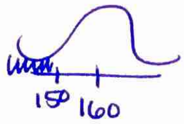
$\mu = 210$
$\sigma = 8$

. Monster energy drinks contain mean of 160 mg of caffeine per can, with a standard deviation of 3 mg per can. Use this scenario to draw a labeled sketch and provide the requested information for each scenario below. *Round answers to nearest* ***ten thousandth***.

work: Sketch:

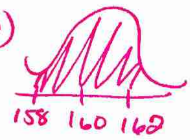
a) probability of a can containing less than 150 mg caffeine: 4.2911×10^{-4}

normcdf(-1E99,150,160,3)



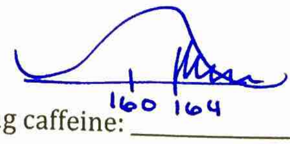
b) probability of a can containing between 158 and 162 mg caffeine: .4950

normcdf(158,162,160,3)



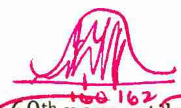
c) probability of a can containing more than 164 mg caffeine: .0912

normcdf(164,1E99,160,3)



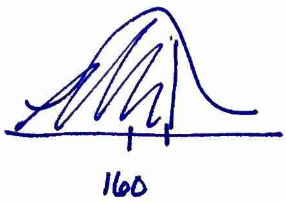
d) **percentile rank** of a can of Monster that contains 162 mg caffeine: .7475

normalcdf(-1E99,162,160,3)



e) How many milligrams of caffeine would represent the 60th percentile? (**invNorm**)

Given percentile



invNorm(.6,160,3)

160.7600

13. Given the list of homemade fish sinker weights (grams), calculate each measure requested, rounding to hundredths where necessary. This data can be assumed to be the entire population of lead weights made on this day.

6.8 8.3 5.0 4.0 7.5 4.0 3.9 5.1 3.3 3.1 3.3 4.7

Mean: 4.92 Std dev: 1.66

Med: 4.35 IQR: 2.35

5.95
−3.6

Q3−Q1

Q1: 3.6

Q3: 5.95

To describe center and spread, which two measures would you use? *Justify your answer.*

Med and IQR

Justification: Skewed

When your data is SYMMETRIC use: mean & Stand dev.

When your data is SKEWED use: Med. & IQR

Skewed Right

14. What does a relatively large standard deviation indicate about a set of data?

data is very spread out

15. The following table shows the number of points scored by two players on the same team over six games.
Use your <u>calculator</u> to answer each of the questions below.

Game	1	2	3	4	5	6
Player 1	8	13	16	10	15	11
Player 2	7	25	6	10	23	7

L1
L2

Provide the mean and standard deviation for each player, to the nearest tenth.

Mean Player 1 12.2

St Dev Player 1 2.8

Mean Player 2 13

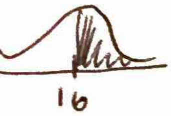
St Dev Player 2 7.9

Which player is a more consistent scorer? Explain your reasoning.

player 1 - smaller SD!

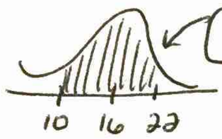
16. A variable is normally distributed with a mean of 16 and a standard deviation of 6. Find the percent of the data set that: *Draw a shaded sketch for each part; label the mean and the end(s) of the shaded area(s).*

(a) is greater than 16



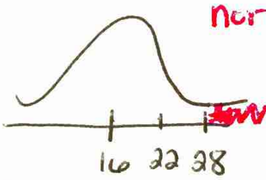
50%

(b) falls between 10 and 22



68%

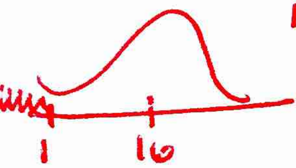
(c) is greater than 28



normalcdf(28, 1E99, 16, 6)

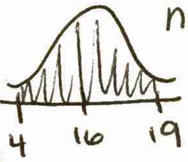
2%

(d) is less than 1



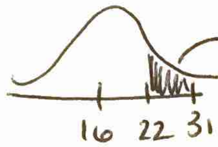
normalcdf(-1E99, 1, 16, 6) = .6%

(e) falls between 4 and 19



normal cdf(4, 19, 16, 6) = 67%

(f) falls between 22 and 31



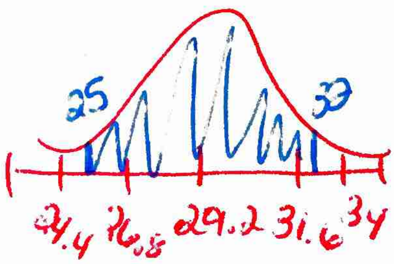
normal cdf(22, 31, 16, 6) = 15%

17. The lengths of full grown sockeye salmon are normally distributed with a mean of 29.2 inches and a standard deviation of 2.4 inches.

(a) Find z-scores for sockeye salmon whose lengths are 25 inches to 32 inches. Round to the nearest hundredth. Show your calculations.

$\frac{25-29.2}{2} = -2.1$ $\qquad$ $\frac{32-29.2}{2} = 1.4$

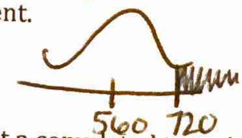
(b) Determine the proportion of the sockeye salmon population, to the nearest percent, that lies between 25 inches and 32 inches. Illustrate your work graphically.



normal cdf(25, 32, 29.2, 2.4) = 84%

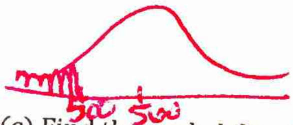
18. Show supporting work for all answers. If the scores on a standardized test are normally distributed with a mean of 560 and a standard deviation of 75:

(a) Find the probability that a test picked at random would have a score larger than 720. Round to the nearest tenth of a percent.



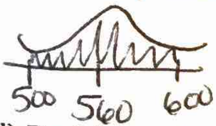
normal cdf [720, 1E99, 560, 75] = 1.6%

(b) Find the probability that a completed test picked at random would have a score less than 500. Round to the nearest tenth of a percent.



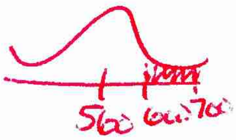
normal cdf [-1E99, 500, 560, 75] = 21.2%

(c) Find the probability that a completed test picked at random would have a score between 500 and 600.



normal cdf [500, 600, 560, 75] = 49.1%

(d) Find the probability that a completed test picked at random would have a score between 600 and 700.

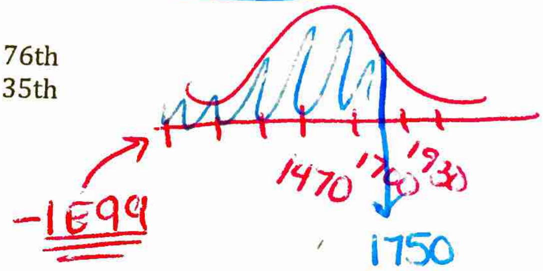


normal cdf [600, 700, 560, 75] → 26.6%

19. The average weight of full grown beef cows is 1470 pounds with a standard deviation of 230 pounds. If the weights are normally distributed, what is the percentile rank of a cow that weighs 1,750 pounds? *Provide a sketch and calculator inputs!*

shade below

(1) 89th
(2) 49th
(3) 76th
(4) 35th



normalcdf [-1E99, 1750, 1470, 230]

20. The prices of the printers in a store have a mean of \$240 and a standard deviation of \$50. The printer that you eventually choose costs \$340.

a) What is the z score for the price of your printer? 2

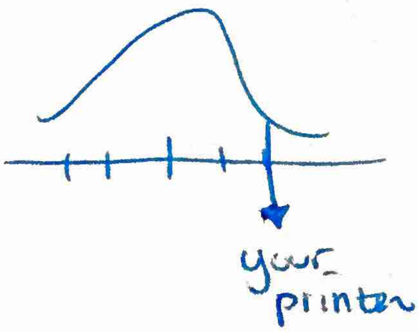
$$z = \frac{340 - 240}{50} = 2$$

b) How many standard deviations above the mean was the price of your printer?

2

c) Explain (as if to someone who hasn't studied probability) what this means in comparison to all other printer prices.

* 97.7% of people have printers this price or cheaper



normal cdf [-1E99, 2, 0, 1]

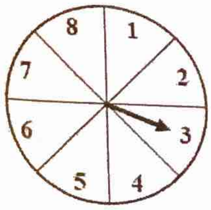
21. A spinner is spun around a circle that is divided up into eight equally sized sectors. Show probability rule calculations to find:

(a) P (Perfect Square | even) → 2, 4, 6, 8

$$\frac{1}{4}$$

(b) P (odd | prime) 2, 3, 5, 7

$$\frac{3}{4}$$



(c) Which is more likely, getting a multiple of four given we spun an even? Or getting an odd, given we spun a number greater than 2? Support your answer with probability rule calculations.

2, 4, 6, 8

P(mult of 4 | even)

$\frac{2}{4} = \frac{1}{2}$

same

=

3, 4, 5, 6, 7, 8

P(odd | >2)

$\frac{1}{2} = \frac{3}{6}$

22. A person flips a coin and notes that it comes up heads. Then the person rolls a standard six-sided die and notes that it comes up as a number less than three.

1. Is the probability that the number came up less than three influenced by the condition of the coin when it is flipped? Explain.

no - mutually exclusive events

2. Use rules to find the probability of flipping heads and then rolling a number less than 3.

→ indep!

P(H) · P(# < 3)

$\left(\frac{1}{2}\right)\left(\frac{2}{6}\right) = \boxed{\frac{1}{6}}$

Provide probability answers rounded to thousandths for all remaining problems, unless otherwise instructed.

23. There is a 34% chance that a person picked at random from the adult population is regular smoker of cigarettes and an 18% chance that a person picked has emphysema. If the percent of the adult population that are both regular smokers and suffer from emphysema is 14%, is being a smoker independent from having emphysema? Justify your result using probability rules, a Venn Diagram, or a hypothetical 1000 table.

P(A) vs P(A | B)

.34 $\frac{.14}{.18}$

.34 ≠ .77

P(A∩B) vs P(A)·P(B)

.14 (.34)(.18)

.14 ≠ .0612

Not indep

24. When an avocado is selected at random from those delivered to a food store, the probability that it is ripe is 0.12, the probability that it is bruised is 0.054, and the probability that it is ripe and bruised is 0.019.

a) Complete the hypothetical 1000 table given below, and use it to find the probability that a randomly selected avocado is bruised given that it is not ripe. (Round your answer to the nearest thousandth.)

	Ripe	Not Ripe	Total
Bruised	19	35	54
Not bruised	101	845	946
Total	120	880	1000

b) Determine whether or not being bruised and ripe are independent events. Justify your answer mathematically.

$P(R \cap B)$ vs $P(R) \cdot P(B)$ | $P(A)$ vs $P(B|R)$

$.019$ $(.12)(.054)$ | $.054$ $\neq$ $\frac{19}{120}$

$\neq .00648$ not indep

$.158$

25. Find to the nearest tenth:

mean 30.2

median 30

mode 35

range 40

stdev 9.8

min 5

Q1 25

Q2 30

Q3 35

max 45

IQR

Q3 - Q1

10

Score	Frequency
5	3
15	7
20	6
25	8
30	15
35	20
40	9
45	7
	n =

Range