## Unit 9 - Radicals and Applying Similarity to Right Triangles

| Day | Classwork | Day | Homework |
| :---: | :---: | :---: | :---: |
| Tuesday $1 / 2$ | Programming Activity | 0 |  |
| Wednesday $1 / 3$ | Review of Radicals | 1 | HW 9.1 |
| Thursday $1 / 4$ | Special Relationships within Right Triangles (Altitude Drawn to the Hypotenuse) | 2 | HW 9.2 |
| Friday $1 / 5$ | Pythagorean Theorem Unit 9 Quiz 1 | 3 | HW 9.3 |
| Monday $1 / 8$ | Special Right Triangles (45-45-90 and 30-60-90) | 4 | HW 9.4 |
| Tuesday $1 / 9$ | Review <br> Unit 9 Quiz 2 | 5 | Review Sheet |
| Wednesday $1 / 10$ | Review | 6 | Study |
| Thursday $1 / 11$ | Unit 9 Test | 7 | Midterm Review \#1 |
| $\begin{gathered} 1 / 12- \\ 1 / 19 \end{gathered}$ | Midterm Review |  |  |

## Simplifying Radicals

A rational number is a number that $\qquad$
An irrational number is a number that $\qquad$
Radicals (like fractions) must always be reduced.
A radical is in simplest form when $\qquad$
1.) $\sqrt{8}$
2.) $\sqrt{54}$
3.) $\sqrt{48}$
4.) $3 \sqrt{200}$
5.) $\frac{3}{4} \sqrt{80}$
6) $\frac{3}{2} \sqrt{28}$

## Adding and Subtracting Radicals

*Radicals must have a common radicand and index to be added or subtracted.
1.) Simplify all radicals, if possible, to determine if the terms have a common radicand.
2.) Combine terms with common radicands by adding or subtracting the coefficients.
1.) $7 \sqrt{5}+3 \sqrt{5}$
2.) $4 \sqrt{3}-\sqrt{3}$
3.) $\sqrt{2}+\sqrt{8}$
4.) $\sqrt{20}-2 \sqrt{5}$

## Multiplying and Dividing Radicals

## Steps:

1.) Multiply or divide using the following rules:

$$
a \sqrt{b} \cdot c \sqrt{d}=a c \sqrt{b d} \quad \frac{a \sqrt{c}}{b \sqrt{d}}=\frac{a}{b} \sqrt{\frac{c}{d}}
$$

2.) Simplify radical, if possible.
1.) $\sqrt{5} \cdot \sqrt{20}$
2.) $-6 \sqrt{2} \cdot 5 \sqrt{8}$
3.) $9 \sqrt{10 x} \cdot \sqrt{30 x^{2}}$
4.) $\frac{2}{5} \sqrt{20}\left(\frac{3}{4} \sqrt{10}\right)$
5.) $5(3+\sqrt{3})$
6.) $2 \sqrt{5}(2 \sqrt{5}+3)$
7.) $7 \sqrt{2}(\sqrt{8}+2 \sqrt{20})$
8.) $(2-\sqrt{3})(4+\sqrt{3})$
9.) $(3-\sqrt{5})^{2}$
10.) $(\sqrt{2}+\sqrt{7})(\sqrt{2}-\sqrt{7})$
11.) $\sqrt{72} \div \sqrt{2}$
12.) $8 \sqrt{48} \div 2 \sqrt{3}$
13.) $\frac{25 \sqrt{24}}{-5 \sqrt{2}}$

## Rationalizing Denominators

*A fraction is not considered simplified if there is a radical in the denominator.
*To rationalize the denominator of a fraction means to find an equivalent fraction in which the denominator is a rational number.
*Steps to rationalize a monomial denominator:
1.) Multiply the numerator and denominator of the fraction by the radical in the denominator to keep a perfect square in the denominator.
2.) Simplify the fraction, which will change the denominator to a rational number.

1) $\frac{2}{\sqrt{5}}$
2) $\frac{4}{\sqrt{18}}$
3) $\frac{3}{2 \sqrt{3 x}}$

$$
\frac{3}{2 \sqrt{3 x}}
$$

$$
\text { 4) } \frac{3 \sqrt{50}}{4 \sqrt{8}}
$$

## Area and Perimeter with Radicals

1.) What is the perimeter of the triangle shown below?

2.) Determine the area and perimeter of the triangle shown.


## Altitude Drawn to the Hypotenuse in a Right Triangle

| Theorem |  |  |
| :---: | :---: | :---: |
| Words | Example | Figures |
| If the altitude is drawn to the <br> hypotenuse of a right triangle, then the <br> two triangles formed are similar to the <br> original triangle and to each other. |  |  |

## Examples

Write a similarity statement identifying the three similar right triangles in the figure.
1.

2.

3.



| Right Triangle Geometric Mean Theorems |  |  |  |
| :---: | :---: | :---: | :---: |
| Theorem | Words | Example | Figures |
| Geometric <br> Mean <br> (Altitude) <br> Theorem <br> (PAP) | The altitude drawn to the <br> hypotenuse of a right triangle <br> separates the hypotenuse into two <br> segments. The length of this <br> altitude is the geometric mean <br> between the lengths of these two <br> segments. |  |  |


| Geometric <br> Mean (Leg) <br> Theorem <br> (HELP) | The altitude drawn to the hypotenuse of a right triangle separates the hypotenuse into two segments. The length of a leg of this triangle is the geometric mean between the length of the hypotenuse and the segment of the hypotenuse adjacent to the leg. |  |  |
| :---: | :---: | :---: | :---: |
| (HALL) | In a right triangle, the product of the hypotenuse and the altitude equals the product of the lengths of the two legs. |  |  |

## Examples

Find $x, y$, and $z$ for each of the following.
1.

2.

3.


## More Practice

1.) Find $A B$ :

2.) Solve for $x$ :

3.) Solve for $x$ and $y$ :


## Pythagorean Theorem

*Only works for right triangles.
*The longest side, called the hypotenuse (c), can be found across from the right angle.

$$
a^{2}+b^{2}=c^{2}
$$

a

1.) The sides of a triangle measure $\sqrt{7}, 2 \sqrt{6}$, and $\sqrt{31}$. Is it a right triangle?
2.) The perimeter of a square is 16 . Find the length of the diagonal of the square.
3.) The side of a rhombus measures 10 and its shorter diagonal is 12 .

Find the length of the longer diagonal.
4.) The length of a rectangle is 7 more than the width. The diagonal is 8 more than the width. Find the dimensions of the rectangle.
5.) Determine the exact area of the shaded region shown.

6.) Prove the Pythagorean Theorem using similar triangles. Provide a well-labeled diagram to support your justification.

## SPECIAL RIGHT TRIANGLES

| $45^{\circ}-\mathbf{4 5 ^ { \circ }} \mathbf{- 9 0 ^ { \circ }}$ Triangle Theorem |  |  |
| :---: | :---: | :---: |
| Words | Example | Figure |
| In a $45^{\circ}-45^{\circ}-90^{\circ}$ triangle, the <br> legs $l$ are congruent and the <br> length of the hypotenuse $h$ is $\sqrt{2}$ <br> times the length of the leg. |  | $C$ |

## Finding the Hypotenuse length

Find $x$.
a.

b.

C.

d.


## Finding the Leg length

Find $x$.
e.

f.

g.


| $30^{\circ}-60^{\circ}-90^{\circ}$ Triangle Theorem |  |  |
| :---: | :---: | :---: |
| Words | Example | Figure |
| In a $30^{\circ}-60^{\circ}-90^{\circ}$ triangle, the length of the hypotenuse $h$ is 2 times the length of the shorter $\operatorname{leg} s$, and the length of the longer $\operatorname{leg} l$ is $\sqrt{3}$ times the length of the shorter leg. |  |  |

## Examples

Find $x$ and $y$.
a.

b.

c.

d.

e.

f.


## More Practice with Special Right Triangles

1) Find the length of a side of an equilateral triangle if its altitude is $7 \sqrt{3}$.
2) Find the altitude of an isosceles triangle if its vertex angle is 120 and its legs measure 8 .
3) In a rhombus with a $60^{\circ}$ angle, one side measures 12. Find the length of both diagonals.
4) Find all missing segments:

$$
\begin{aligned}
& \mathrm{AD}= \\
& \mathrm{CD}= \\
& \mathrm{BC}= \\
& \mathrm{BD}= \\
& \mathrm{AB}=
\end{aligned}
$$


5) Find the perimeter of a square whose diagonal is $9 \sqrt{2}$.
6) In an isosceles triangle with a base angle of 45 , one leg measures $5 \sqrt{2}$. Find the altitude.
7) If $P T=8$, find all missing segments:

PS =
ST =
RT =
RS =


