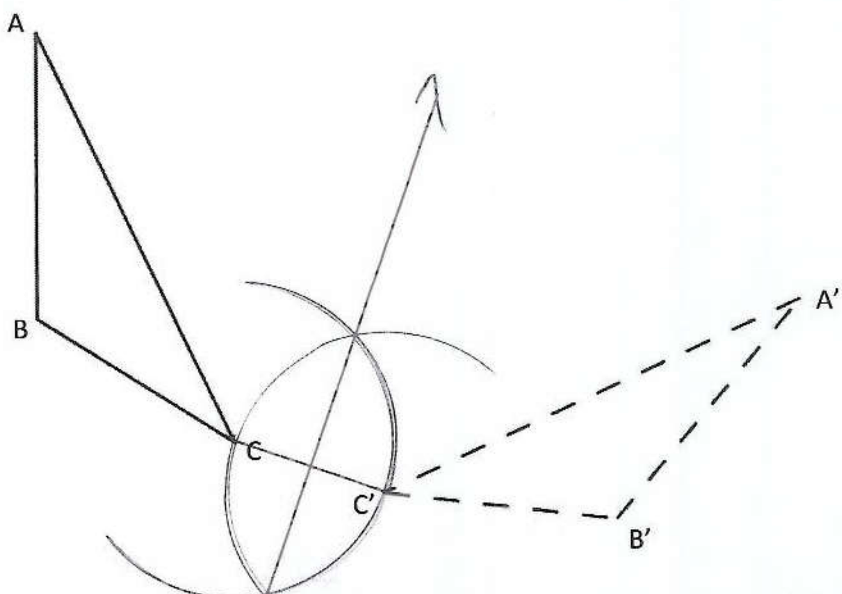
Perpendicular Bisector Relationship to Transformations

Review of perpendicular bisector:

- Two lines are perpendicular if they ___intersect___, and if any of the angles formed by the intersection of the lines is a ___right (90°)___ angle.
- Two segments are perpendicular if the lines containing them are ___perpendic___.
- As you have learned previously, the perpendicular bisector is also known as the ___line of reflection___

Reflections and Perpendicular Bisectors:

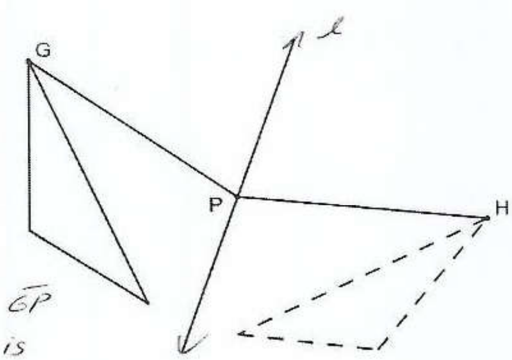
Draw the line of reflection which maps ΔABC to $\Delta A'B'C'$



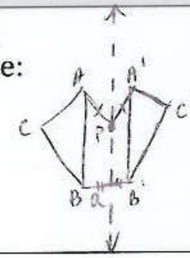
We have determined that any point on the pre-image figure is the ___same distance___ from the line of reflection as its image. Therefore, the two points are ___equidistant___ from the point at which the line of reflection (*perpendicular bisector*) intersects the segment connecting the pre-image point to its image.

What about other points on the perpendicular bisector? Are they also equidistant from the pre-image and image points as seen in the image to the right?

(In other words, is $GP = HP$?)



Justify your conclusion: Yes! If a segment such as $\overline{GP}$ is reflected over line ℓ then distance is preserved. Therefore since $\Lambda(\overline{GP}) = \overline{HP}$ ~~or~~, $GP = HP$

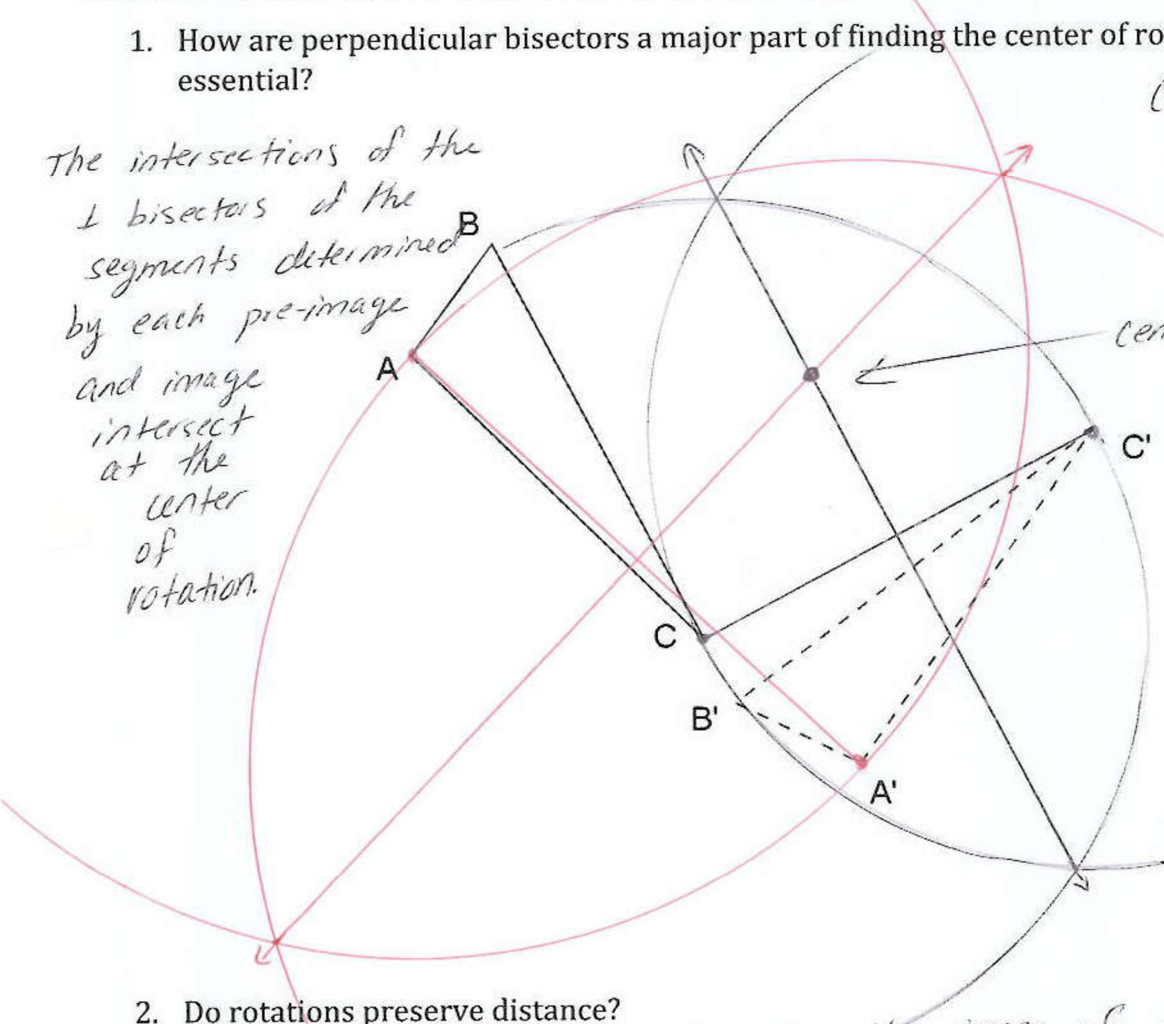
Summary	
Every point on the line of reflection (perpendicular bisector) is equidistant from a pre-image and its corresponding image.	Example:  AP = P'P BQ = B'Q

Rotations and Perpendicular Bisectors:

Review: Find the center of rotation for the transformation below.

1. How are perpendicular bisectors a major part of finding the center of rotation? Why are they essential?

The intersections of the ⊥ bisectors of the segments determined by each pre-image and image intersect at the center of rotation.



Connect a point on the pre-image to its image. ~~Draw~~ Construct the ⊥ bisector of this segment. Repeat with another pair of points. The intersection of the ⊥ bisectors is the center of rotation.

2. Do rotations preserve distance?

Yes the distance between the image of 2 pts is always equal to the distance between the original 2 points.

3. Which of the statements below is true of the distances in the figure? Justify your response.

a. $AB = A'B'$

True. corresponding sides of the pre-image and image are equal in length.

b. $AA' = BB'$

False. These distances/lengths ~~are twice the distance~~ don't represent sides of the ~~from the orig. point to the~~ original figure or image.

Translations and Perpendicular Bisectors:

Translations involve constructing parallel lines (which certainly can be done by constructing perpendiculars, but perpendicular bisectors are **<u>not essential</u>** to constructing translations).

Examples:

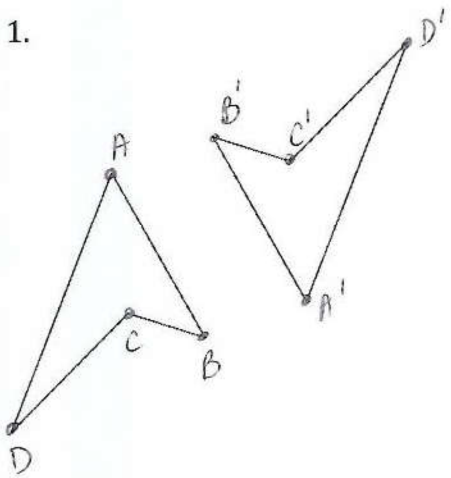
For #1-4, in each pre-image/image combination below:

(a) Identify the type of transformation

(b) State whether perpendicular bisectors play a role in constructing the transformation and, if so, what role

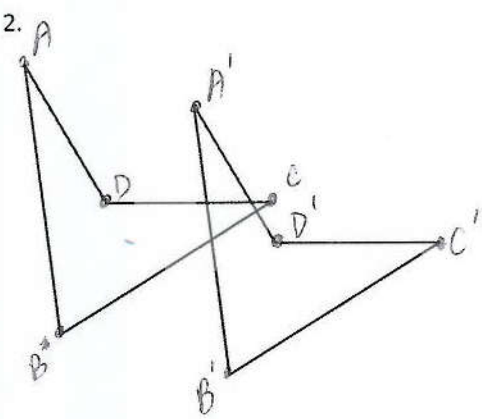
(c) Identify two congruent segments from the pre-image to the image. For the last requirement, you will have to label vertices on the pre-image and image.

1.



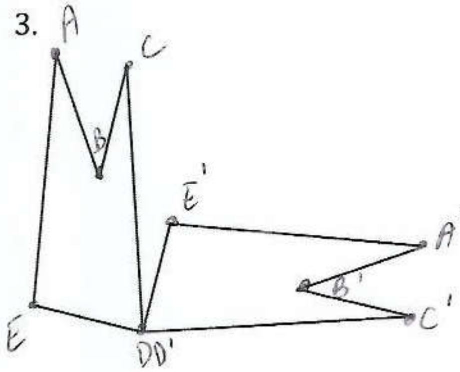
Transformation	Perpendicular bisectors?	Examples of distance preservation
Rotation	yes Find the center of rotation	AB = A'B' BC = B'C' CD = C'D' AD = A'D'

2.



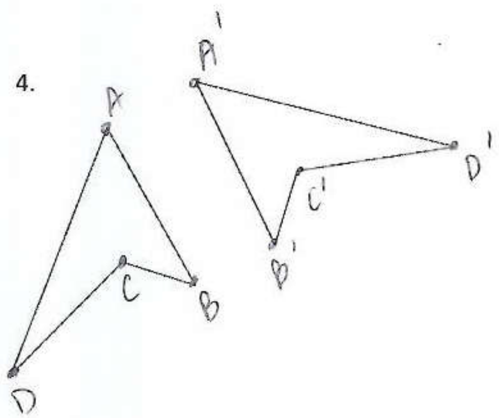
Transformation	Perpendicular bisectors?	Examples of distance preservation
Translation	Not necessary	AB = A'B' AD = A'D' BC = B'C' CD = C'D'

3.



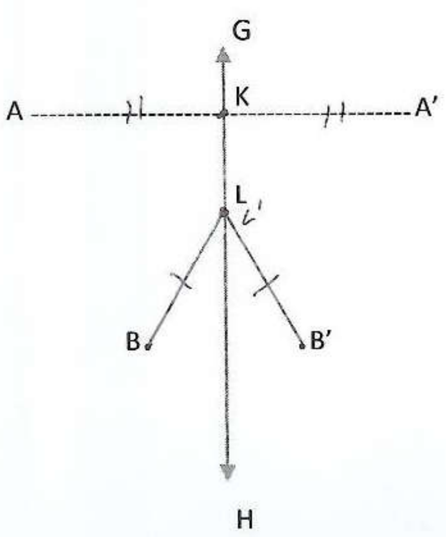
Transformation	Perpendicular bisectors?	Examples of distance preservation
Rotation	yes Find the center of Rotation	$AB = A'B'$ $BC = B'C'$ $CD = C'D'$ $DE = D'E'$ $AD = A'D'$

4.



Transformation	Perpendicular bisectors?	Examples of distance preservation
Reflection	yes Finding the line of reflection	$AB = A'B'$ $BC = B'C'$ $CD = C'D'$ $AD = A'D'$

5. In the figure below, $\overleftrightarrow{GH}$ is the line of reflection. State and justify two conclusions about distances in this figure. At least one of your statements should refer to perpendicular bisectors.



$\left.\begin{array}{l} AK = A'K \\ BL = B'L' \end{array}\right\}$ every pre-image and its corresponding image are equidistant from the line of reflection, which is the ⊥ bisector.

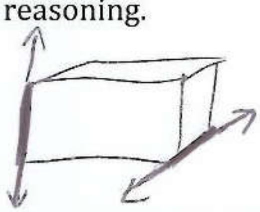
$AB = A'B'$ a reflection preserves distance. so the distance between 2 pts of the pre-image will equal ~~that of~~ the distance between the corresp. pts of the image.

Construct Parallel Lines Through Rotations

Day 2 Objective: Use rigid motions (rotations) to verify the parallel postulate.

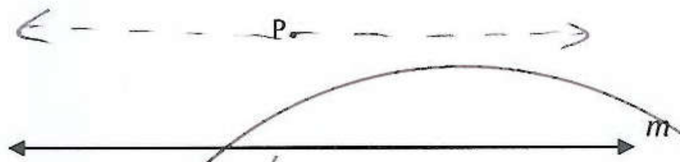
Recall that two lines are *parallel* if they lie in the <u>same plane</u> and do not <u>intersect</u>.

1. Why is the phrase *in the plane* critical to the definition of parallel lines? Explain or illustrate your reasoning.



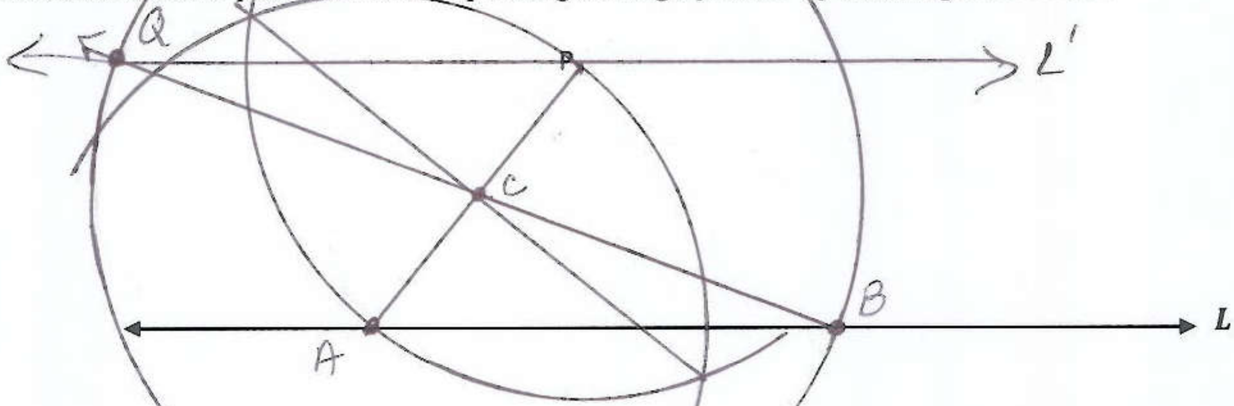
Two lines in different planes may be skew lines (never intersect but are not ∥)

2. In the diagram below, how many lines can be drawn through point P that are parallel to the given line m?



only one!

3. Construct a line parallel to line L passing through point P by rotating line L 180°.



a. Label any point A on L.

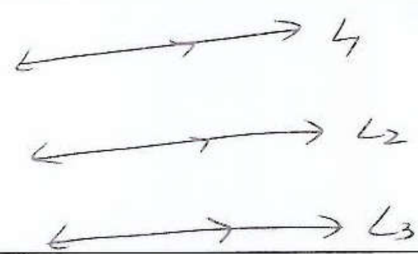
b. Find the midpoint, C, of segment AP by constructing the perpendicular bisector.

c. Perform a 180° rotation around center C using the following steps.
   i. Pick another point B on L.
   ii. Draw line $\overleftrightarrow{CB}$.
   iii. Draw circle: center C, radius CB.
   iv. Label the other point where the circle intersects $\overleftrightarrow{CB}$ by Q.
   v. Draw line $\overleftrightarrow{PQ}$.

d. Label the image of the rotation by 180° of L by $L' = R_{C,180}(L)$.

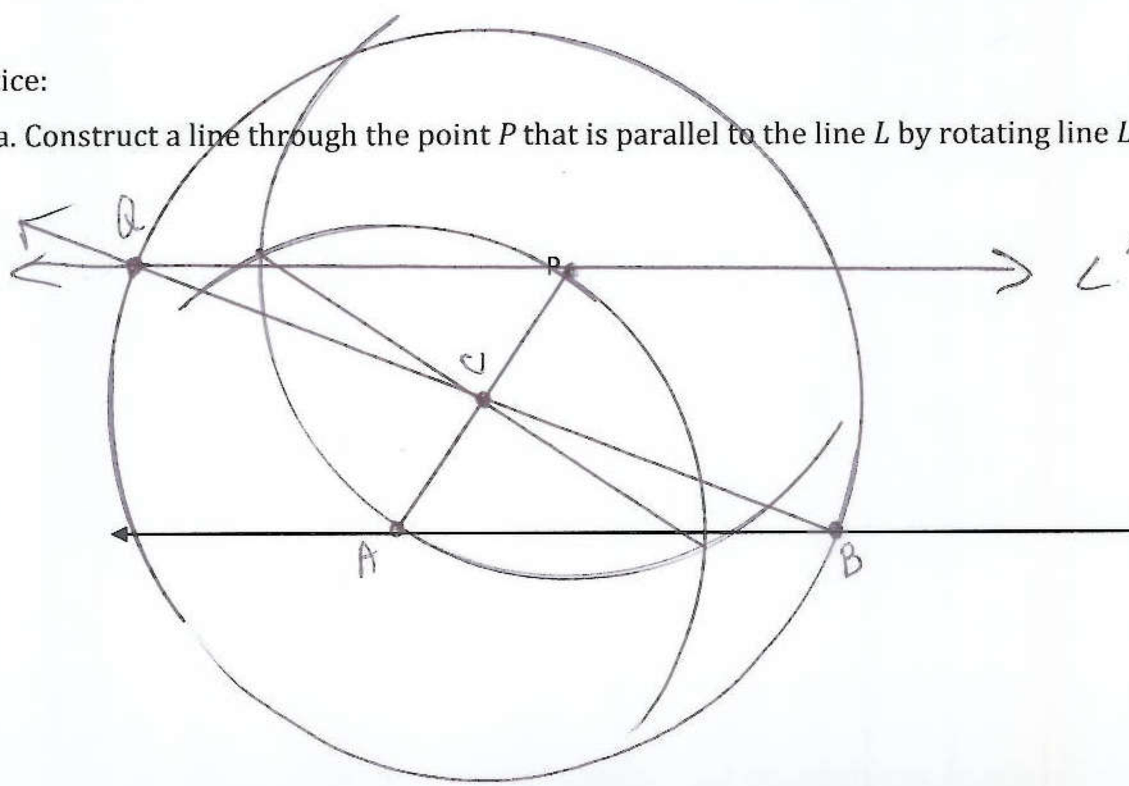
4. Explain why this rotation results in line L parallel to line L'.

[Teacher note: see either indirect proof on page 134 or alternate interior angle justification on page 135. This could also be justified using corresponding angles]

Summary	
Parallel Postulate: Through a given external pt point there is at most 1 line parallel to a given line	Example: 
Theorem: If three distinct lines L_1, L_2, and L_3 in the plane have the property that $L_1 \parallel L_2$ and $L_2 \parallel L_3$, then $L_1 \parallel L_3$ (Abbreviated: $\parallel$-transitivity).	

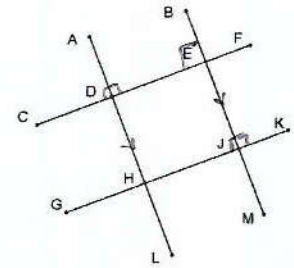
Practice:

1a. Construct a line through the point P that is parallel to the line L by rotating line L 180°.



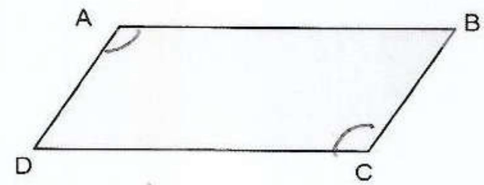
Practice Exercises

1. In the figure, $\overline{AL} \parallel \overline{BM}$, $\overline{AL} \perp \overline{CF}$, and $\overline{GK} \perp \overline{BM}$. Prove that $\overline{CF} \parallel \overline{GK}$.



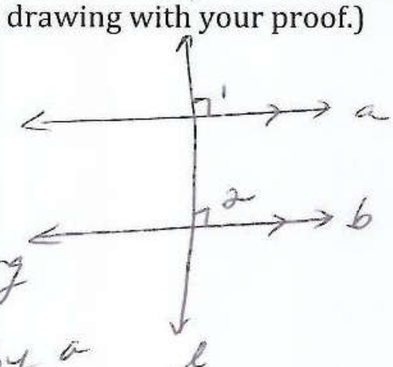
Statements	Reasons
① $\overline{AL} \parallel \overline{BM}$, $\overline{AL} \perp \overline{CF}$, $\overline{GK} \perp \overline{BM}$	① Given
② $\angle ADC$, $\angle BJH$ are rt. $\angle$'s	② $\perp$ lines intersect forming right $\angle$'s
③ $\angle ADC \cong \angle BJH$	③ All right $\angle$'s are $\cong$
④ $\angle ADC \cong \angle BED$	④ if 2 $\parallel$ lines are cut by a transv. corresp. $\angle$'s are $\cong$
⑤ $\angle BJH \cong \angle BED$	⑤ substitution
⑥ $\overline{CF} \parallel \overline{GK}$	⑥ if corresp. $\angle$'s are $\cong$, then the lines are $\parallel$.

2. Given that $\angle B$ and $\angle C$ are supplementary and $\angle A \cong \angle C$, prove that $\overline{AD} \parallel \overline{BC}$.



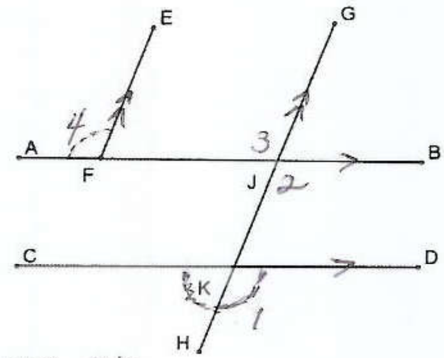
Statements	Reasons
① $\angle B$, $\angle C$ are supp. $\angle A \cong \angle C$	① Given
② $m\angle B + m\angle C = 180°$	② Definition of supp. $\angle$'s (2 $\angle$'s that are supp. add to 180°)
③ $m\angle A + m\angle B = 180°$	③ substitution
④ $\overline{AD} \parallel \overline{BC}$	④ if consecutive interior $\angle$'s are supp. then the lines are $\parallel$.

3. Mathematicians state that *if a transversal to two parallel lines is perpendicular to one of the lines, then it is perpendicular to the other.* Prove this statement. (Include a labeled drawing with your proof.)



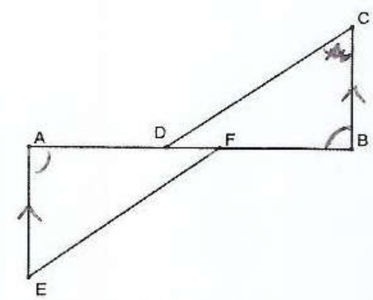
Statements	Reasons
① $a \parallel b$, $l \perp a$	① Given
② $\angle 1$ is a rt $\angle$	② $\perp$ lines intersect forming rt. $\angle$'s
③ $\angle 1 \cong \angle 2$	③ if 2 $\parallel$ lines are cut by a transv. then corresp. $\angle$'s are $\cong$
④ $\angle 2$ is a rt. $\angle$	④ substitution
⑤ $l \perp b$	⑤ $\perp$ lines intersect forming rt. $\angle$'s

4. In the figure, $\overline{AB} \parallel \overline{CD}$ and $\overline{EF} \parallel \overline{GH}$. Prove that ∠AFE ≅ ∠DKH.



STATEMENTS	REASONS
① $\overline{AB} \parallel \overline{CD}$, $\overline{EF} \parallel \overline{GH}$	① Given
② ∡1 ≅ ∡2 ∡3 ≅ ∡4	② if 2 ∥ lines are cut by a transv. then corresp. ∡'s are ≅
③ ∡2 ≅ ∡3	③ vertical ∡'s are ≅
④ ∡1 ≅ ∡3 (∡AFE ≅ ∡DKH)	substitution

5. In the figure, ∠E and ∠AFE are complementary and ∠C and ∠BDC are complementary. Prove that $\overline{AE} \parallel \overline{CB}$.



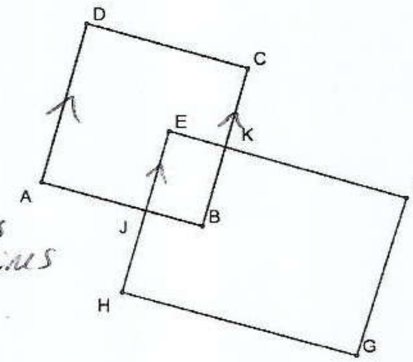
STATEMENTS	REASONS
① ∡E, ∡AFE are comp. ∡C, ∡BDC are comp.	① Given
② m∡E + m∡AFE = 90° m∡C + m∡BDC = 90°	② the sum of the measures of 2 ∡'s that are complem. is 90°
③ m∡E + m∡AFE + m∡A = 180° m∡C + m∡BDC + m∡B = 180°	③ the sum of the interior ∡'s of a Δ is 180°
④ 90 + m∡A = 180° 90 + m∡B = 180°	④ substitution
⑤ 90 + m∡A = 90 + m∡B	⑤ substitution
⑥ m∡A = m∡B	⑥ subtraction
⑦ $\overline{AE} \parallel \overline{CB}$	⑦ if alt. int prop. ∡'s are ≅ then the lines are ∥.

6. Given a line L and a point P not on the line, the following directions can be used to draw a perpendicular M to the line L through the point P based upon a rotation by 180°:

   a. Pick and label a point A on the line L so that the circle (center P, radius AP) intersects L twice.
   b. Use a protractor to draw a perpendicular line N through the point A (by constructing a 90° angle).
   c. Use the directions in Example 2 to construct a parallel line M through the line P.

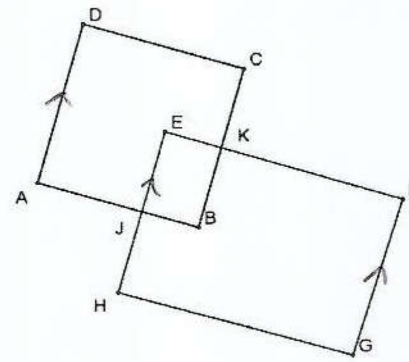
Do the construction. Why is the line M perpendicular to the line L in the figure you drew? Why is the line M the only perpendicular line to L through P?

6. $\overline{AD} \parallel \overline{BC}$ and $\angle EJB$ is supplementary to $\angle JBK$. Prove that $\overline{AD} \parallel \overline{JE}$.



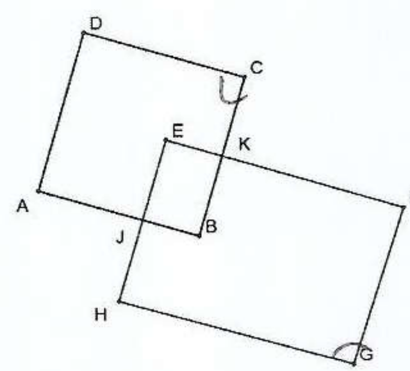
STATEMENTS	REASONS
① $\overline{AD} \parallel \overline{BC}$; $\measuredangle EJB$, $\measuredangle JBK$ are supplem.	① Given
② $\overline{JE} \parallel \overline{BC}$	② if consecutive int. $\measuredangle$'s are supp. then the lines are $\parallel$.
③ $\overline{AD} \parallel \overline{JE}$	③ $\parallel$ transitivity

7. $\overline{AD} \parallel \overline{FG}$ and $\overline{EJ} \parallel \overline{FG}$. Prove that $\angle DAJ$ and $\angle EJA$ are supplementary.



STATEMENTS	REASONS
① $\overline{AD} \parallel \overline{FG}$; $\overline{EJ} \parallel \overline{FG}$	① Given
② $\overline{AD} \parallel \overline{EJ}$	② $\parallel$ transitivity
③ $\measuredangle DAJ$ & $\measuredangle EJA$ are supp	③ if 2 $\parallel$ lines are cut by a transv. consec. int. $\measuredangle$'s are supp.

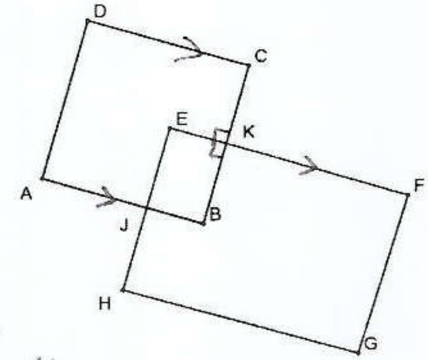
8. $\angle C \cong \angle G$ and $\angle B$ is supplementary to $\angle G$. Prove that $\overline{DC} \parallel \overline{AB}$.



STATEMENTS	REASONS
① $\measuredangle C \cong \measuredangle G$, $\measuredangle B$ & $\measuredangle G$ are supplem.	① Given
② $\measuredangle C$ & $\measuredangle B$ are supplem.	② substitution
③ $\overline{DC} \parallel \overline{AB}$	③ if consecutive interior $\measuredangle$'s are supp. then the lines are $\parallel$.

$\overline{CB}$

9. $\overline{AB} \parallel \overline{EF}, \overline{EF} \perp \overline{CD},$ and $\angle$EKC is supplementary to $\angle$KCD. Prove that $\overline{AB} \parallel \overline{DC}$.



STATEMENTS	REASONS
① $\overline{AB} \parallel \overline{EF}$, $\measuredangle$EKC is supp. to $\measuredangle$KCD	① Given
② $\overline{CD} \parallel \overline{EF}$	② if consec. int $\measuredangle$'s are supp. then the lines are $\parallel$.
③ $\overline{AB} \parallel \overline{CD}$	③ $\parallel$ transitivity

Sequence of Rigid Motions and Composition of Transformations

1. List all rigid motions we have studied so far.

   rotations
   reflections
   translations

2. A ___Symmetry___ is a rigid motion that carries a figure to itself.

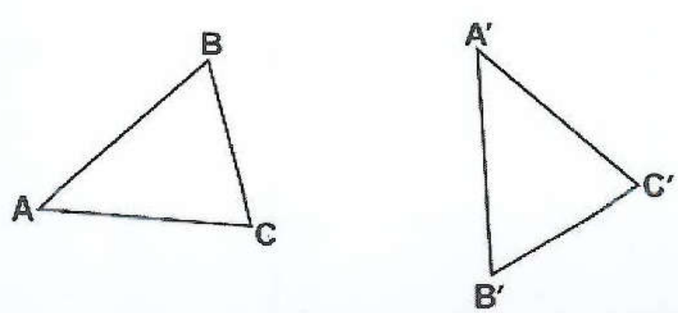
3. Rigid motions preserve ___distance / lengths___ and ___$\measuredangle$ measures___.

4. A ___correspondence___ between two figures is a pairing of each vertex of one figure with one and only one vertex of another figure. A congruence creates a correspondence because

   ___every pre-image maps to an image___

   ____

4a. $\triangle ABC \cong \triangle A'B'C'$ as a result of a two rigid motions. List the corresponding vertices.

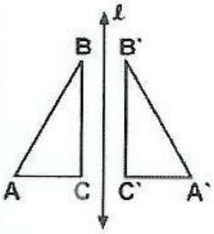


A, A'
B, B'
C, C'

For #5-8,

a. Identify the rigid motion used to map the pre-image on to the image.
b. Identify the corresponding angles and sides

5.



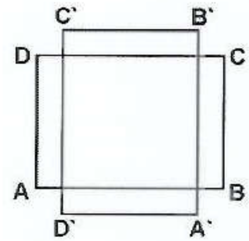
reflection Δ_ℓ

$\Delta_\ell(A) = A'$ $\quad \Delta_\ell(\overline{AB}) = \overline{A'B'}$

$\Delta_\ell(B) = B'$ $\quad \Delta_\ell(\overline{BC}) = \overline{B'C'}$

$\Delta_\ell(C) = C'$ $\quad \Delta_\ell(\overline{AC}) = \overline{A'C'}$

6.



$R_{90°}$

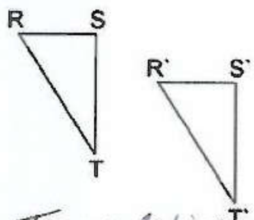
$R_{90°}(B) = B'$ $\quad R_{90°}(\overline{AB}) = \overline{A'B'}$

$R_{90°}(A) = A'$ $\quad R_{90°}(\overline{BC}) = \overline{B'C'}$

$R_{90°}(C) = C'$ $\quad R_{90°}(\overline{CD}) = \overline{C'D'}$

$R_{90°}(D) = D'$ $\quad R_{90°}(\overline{AD}) = \overline{A'D'}$

7.



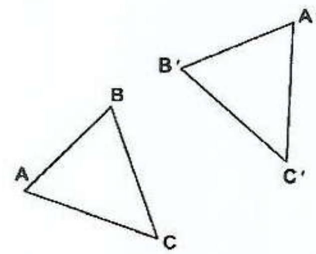
Translation

$T(R) = R'$ $\quad T(\overline{RS}) = \overline{R'S'}$

$T(S) = S'$ $\quad T(\overline{ST}) = \overline{S'T'}$

$T(T) = T'$ $\quad T(\overline{TR}) = \overline{T'R'}$

8.



reflection

$A \to A'$ $\quad \overline{AB} \to \overline{A'B'}$

$B \to B'$ $\quad \overline{BC} \to \overline{B'C'}$

$C \to C'$ $\quad \overline{AC} \to \overline{A'C'}$

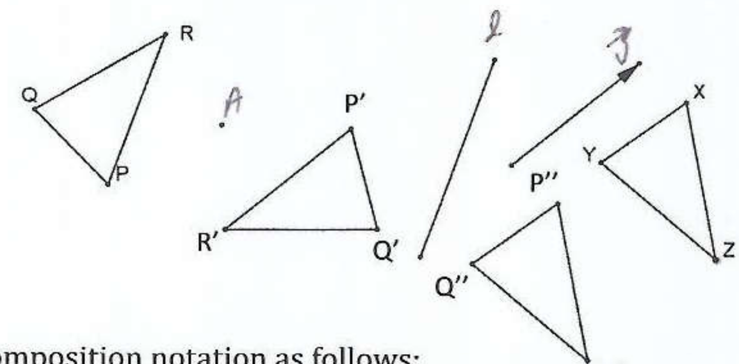
Sometimes there is more than one rigid motion needed to map one figure on to another. This is called a **composition** of transformations. For example the diagram below shows a composition of three rigid motions which map ΔQPR to ΔYXZ.

Identify the three rigid motions used in the order in which they were performed:

ΔQPR to $\Delta Q'P'R'$: Rotation

$\Delta Q'P'R'$ to $\Delta Q''P''R''$: Reflection

$\Delta Q''P''R''$ to ΔYXZ: Translation



This series of rigid motions can be described in composition notation as follows:

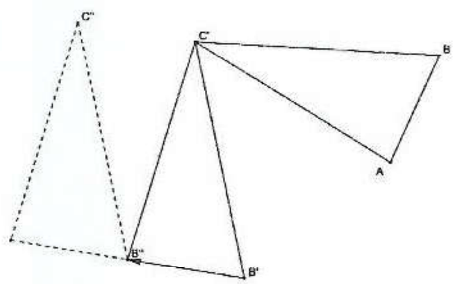
$T_{\vec{v}}(\Delta_\ell(R_D(\Delta PQR)))$ or $T_{\vec{v}} \circ \Delta_\ell \circ R_D(\Delta PQR)$

Definition of Congruence

Two figures in a plane are congruent if there exists a finite composition of basic rigid motions that map one figure onto the other figure.

For #8-9, write the series of compositions in words and notation.

8.

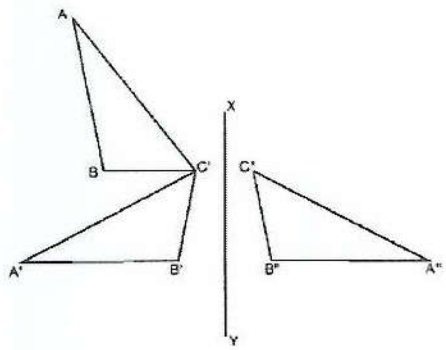


① Rotation about C

② Translation along $B'B''$

$T_{B'B''}(R_C(\Delta ABC))$

9.

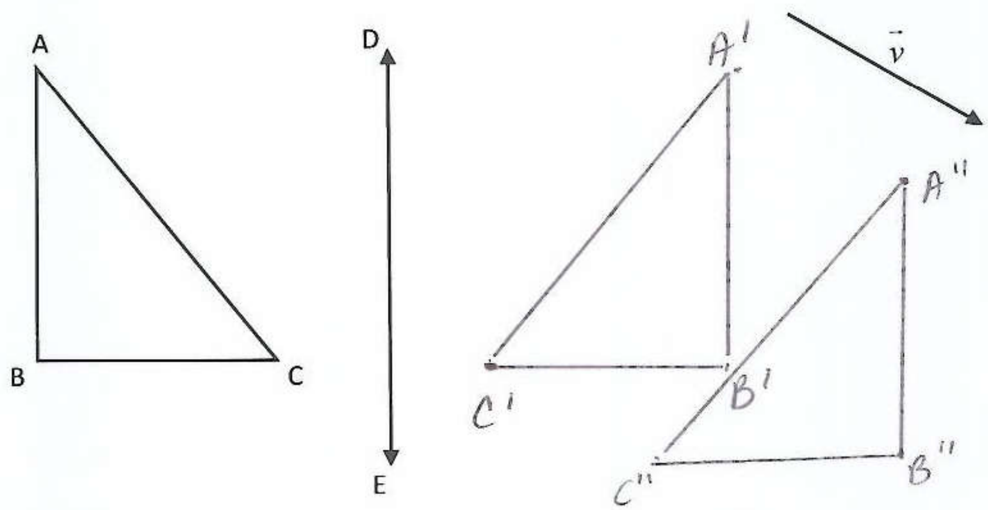


① Rotation about C

② reflection over $\overleftrightarrow{XY}$

$\Lambda_{\overline{XY}}(R_C(\Delta ABC))$

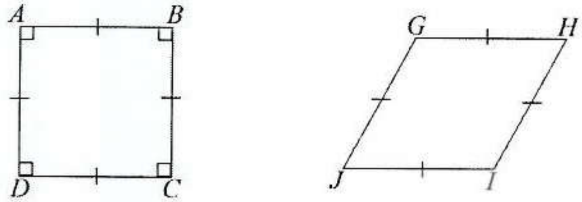
10. Given ΔABC, sketch its image using the composition: $T_{\vec{v}}(\Lambda_{DE}(\Delta ABC))$



10a. Is ΔABC congruent to $\Delta A''B''C''$? Justify using rigid motions.

yes. a reflection and translation preserve distance and ∡ measure so $\Delta ABC \cong \Delta A'B''C''$

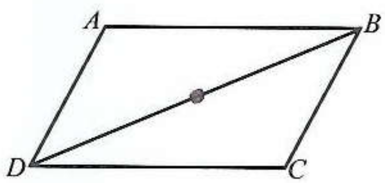
11. Assume the following figures are drawn to scale. Using your understanding of congruence explain why square *ABCD* and rhombus *GHIJ* are not congruent.



In order for the figures to be $\cong$, there must be a rigid motion that maps ABCD to GHIJ. Since rigid motions preserve $\angle$ measure, if $\angle A$ is a rt $\angle$, $\angle G$ would also be a right $\angle$. Since these $\angle$'s are not $\cong$, the figures can't be $\cong$.

12. Use the figures below to answer each question:

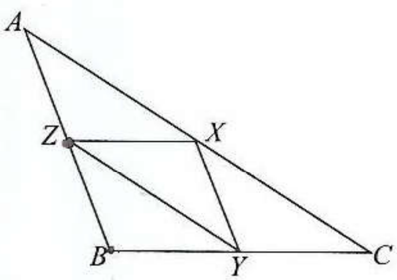
a. $\triangle ABD \cong \triangle CDB$. What rigid motion(s) maps $\overline{CD}$ onto $\overline{AB}$? Find 2 possible solutions.



① A rotation 180° about the midpt of $\overline{BC}$

② A rotation 180° about C followed by a translation along $\overrightarrow{CA}$.

b. All of the smaller sized triangles are congruent to each other. What rigid motion(s) map $\overline{ZB}$ onto $\overline{AZ}$? Find 2 possible solutions.



① $T_{\overrightarrow{ZA}}$

② Rotation 180° about Z followed by a rotation 180° about the midpt. of $\overline{AZ}$

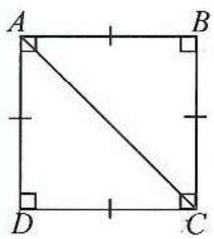
③ 180° rotation about the midpt. of $\overline{ZY}$ followed by a translation along vector $\overrightarrow{YZ}$

Day 5: Summary of Correspondence and Transformations

Day 5 Objective: Apply a sequence of rigid motions from one figure to another in order to demonstrate that the figures are congruent

Recall: If a rigid motion results in every side and every angle of the pre-image mapping onto every corresponding side and angle of the image, we will assert that the triangles are ≅.

1a. $ABCD$ is a square, and $\overline{AC}$ is one diagonal of the square. $\triangle ABC$ is a reflection of $\triangle ADC$ across segment $\overline{AC}$. Complete the table below identifying the missing corresponding angles and sides.



Corresponding angles	Corresponding sides
$\angle BAC \rightarrow \angle DAC$ $\angle ABC \rightarrow \angle ADC$ $\angle BCA \rightarrow \angle DCA$	$\overline{AB} \rightarrow \overline{AD}$ $\overline{BC} \rightarrow \overline{DC}$ $\overline{AC} \rightarrow \overline{AC}$

1b. Are the corresponding sides and angles congruent? Justify your response.

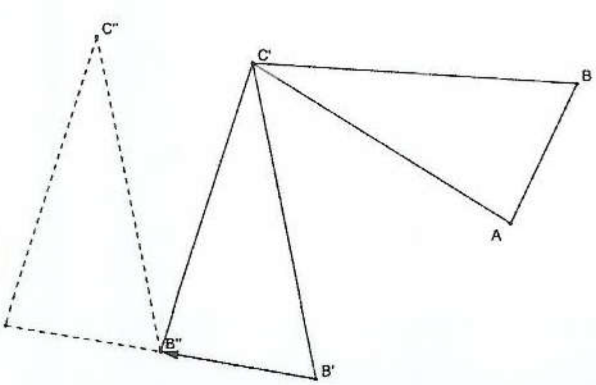
Yes. Since the triangle was reflected over $\overline{AC}$ and a reflection is a rigid motion, then distance and ∡ measure are preserved.

1c. Is $\triangle ABC \cong \triangle ADC$? Justify your response.

Yes. since $\triangle ADC$ is the image of $\triangle ABC$ under a reflection, the △s must be ≅.

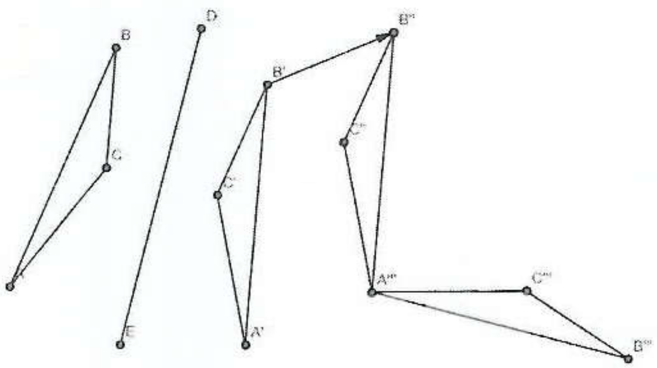
For #2-4, each exercise shows a sequence of rigid motions that map a pre-image onto a final image. Identify each rigid motion in the sequence, writing the composition using function notation. Trace the congruence of each set of corresponding sides and angles through all steps in the sequence, proving that the pre-image is congruent to the final image by showing that every side and every angle in the pre-image maps onto its corresponding side and angle in the image. Finally, make a statement about the congruence of the pre-image and final image.

2.



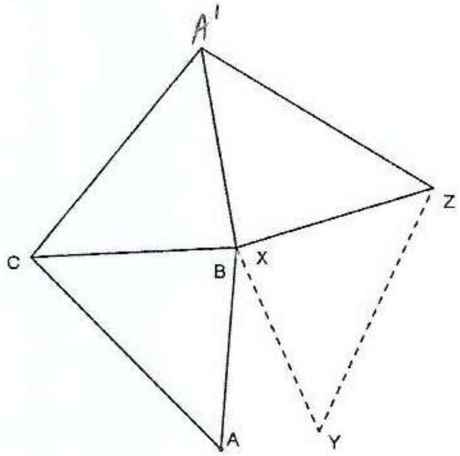
Sequence of rigid motions (2)	Rotation, Translation
Composition in function notation	$T_{B'B''}(R_{C}(\triangle ABC))$
Sequence of corresponding sides	$\overline{AB} \rightarrow \overline{A''B''}$ $\overline{AC} \rightarrow \overline{A''C''}$ $\overline{BC} \rightarrow \overline{B''C''}$
Sequence of corresponding angles	$\angle A \rightarrow \angle A''$ $\angle C \rightarrow \angle C''$ $\angle B \rightarrow \angle B''$
Triangle congruence statement	$\triangle ABC \cong \triangle A''B''C''$

3.



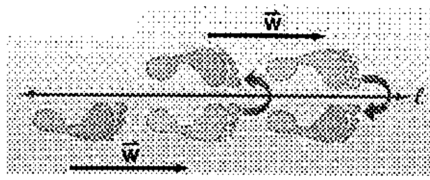
Sequence of rigid motions (3)	reflection, translation, rotation
Composition in function notation	$R_{A'''}(T_{B'B''}(\Delta_{\overline{DE}}(\Delta ABC)))$
Sequence of corresponding sides	$\overline{AB} \rightarrow A'''B'''$ $\overline{AC} \rightarrow A'''C'''$ $\overline{BC} \rightarrow B'''C'''$
Sequence of corresponding angles	$\angle A \rightarrow \angle A'''$ $\angle C \rightarrow \angle C'''$ $\angle B \rightarrow \angle B'''$
Triangle congruence statement	$\Delta ABC \cong \Delta A'''B'''C'''$

4.



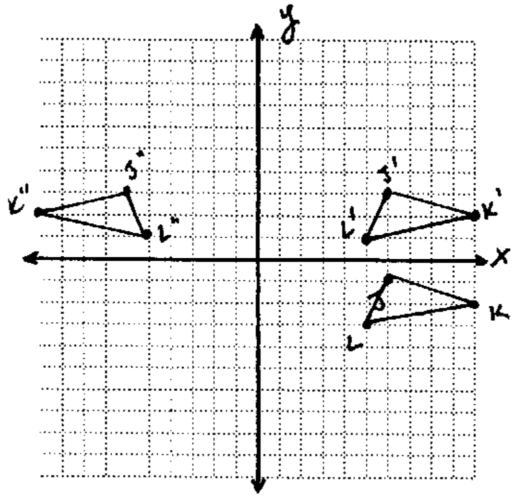
Sequence of rigid motions (3)	reflections (3)
Composition in function notation	$\Delta_{\overline{XZ}}(\Delta_{\overline{BA'}}(\Delta_{\overline{BC}}(\Delta ABC)))$
Sequence of corresponding sides	$\overline{AB} \rightarrow \overline{XY}$ $\overline{AC} \rightarrow \overline{YZ}$ $\overline{BC} \rightarrow \overline{XZ}$
Sequence of corresponding angles	$\angle A \rightarrow \angle Y$ $\angle C \rightarrow \angle Z$ $\angle B \rightarrow \angle X$
Triangle congruence statement	$\Delta ABC \cong \Delta YXZ$

Compositions of Transformations in the Coordinate Plane

Glide Reflection		
Words	Example	
A composition of a translation followed by a reflection in a line parallel to the translation vector.	distance & angle measure are preserved	

Examples

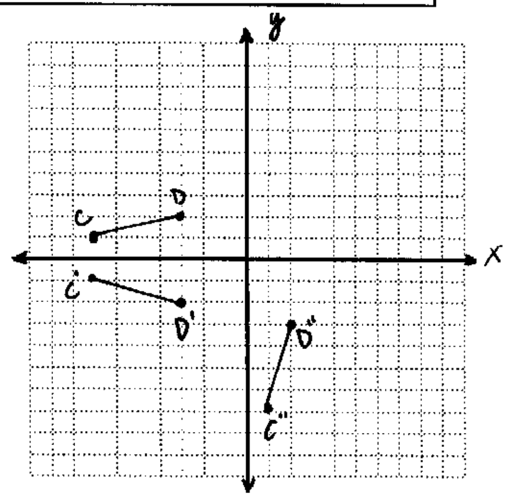
1. Triangle *JKL* has vertices *J*(6, -1), *K*(10, -2), and *L*(5, -3). Graph triangle *JKL* and its image after a translation along <0, 4> and a reflection in the y-axis.



$$J(6,-1) \xrightarrow{T_{0,4}} J'(6,3) \xrightarrow{\Delta_{y\text{-axis}}} J''(-6,3)$$
$$K(10,-2) \longrightarrow K'(10,2) \longrightarrow K''(-10,2)$$
$$L(5,-3) \longrightarrow L'(5,1) \longrightarrow L''(-5,1)$$

Compositions of Rigid Motions
The composition of two (or more) rigid motions is a rigid motion.

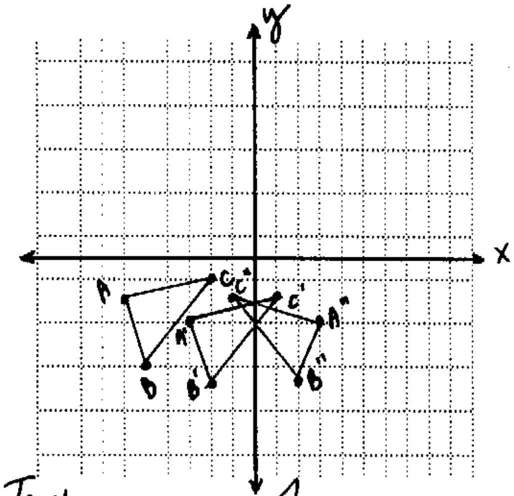
2. The endpoints of $\overline{CD}$ are C (-7, 1) and D (-3, 2). Graph $\overline{CD}$ and its image after a reflection in the *x*-axis followed by a rotation 90 degrees about the origin.



$$C(-7,1) \xrightarrow{\Delta_{x\text{-axis}}} C'(-7,-1) \xrightarrow{R_{0,90^\circ}} C''(1,-7)$$
$$D(-3,2) \longrightarrow D'(-3,-2) \longrightarrow D''(2,-3)$$

3. Triangle ABC has vertices $A(-6, -2)$, $B(-5, -5)$, and $C(-2, -1)$. Graph triangle ABC and its image after the composition of transformations listed below.

a. $\Lambda_{y\text{-}axis} \circ T_{3,-1}(\Delta ABC)$

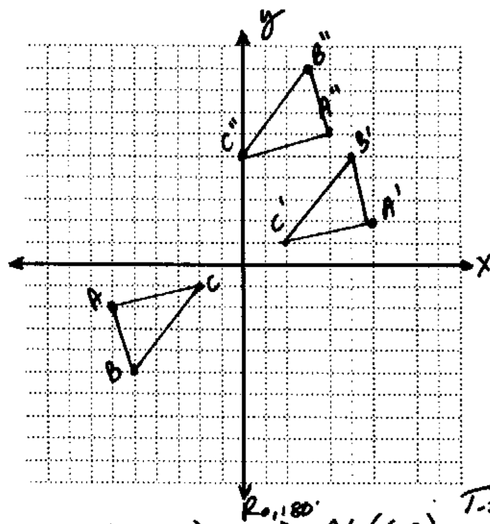


$A(-6,-2) \xrightarrow{T_{3,-1}} A'(-3,-3) \xrightarrow{\Lambda_{y\text{-axis}}} A''(3,-3)$

$B(-5,-5) \rightarrow B'(-2,-6) \rightarrow B''(2,-6)$

$C(-2,-1) \rightarrow C'(1,-2) \rightarrow C''(-1,-2)$

b. $T_{-2,4} \circ R_{(0,0),180°}(\Delta ABC)$



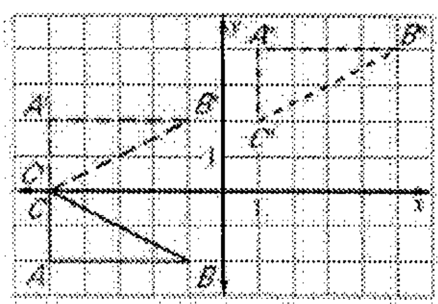
$A(-6,-2) \xrightarrow{R_{0,180°}} A'(6,2) \xrightarrow{T_{-2,4}} A''(4,6)$

$B(-5,-5) \rightarrow B'(5,5) \rightarrow B''(3,9)$

$C(-2,-1) \rightarrow C'(2,1) \rightarrow C''(0,5)$

4. Describe the composition of transformations below in both words and using composition notation.

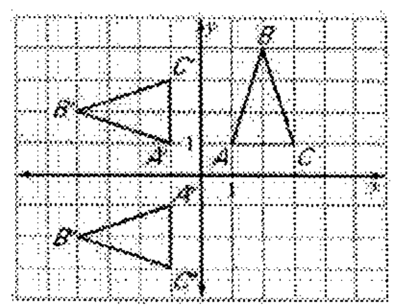
a.



ΔABC was reflected over the x-axis followed by a translation along vector <6,2>

$T_{6,2}(\Lambda_{x\text{-axis}}(\Delta ABC))$

b.

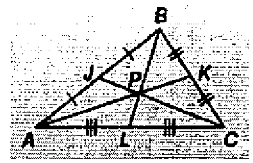


ΔABC was rotated 90° about the origin followed by a reflection over the x-axis

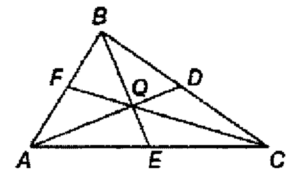
$\Lambda_{x\text{-axis}} \circ R_{0,90°}(\Delta ABC)$

TRIANGLE CENTERS [4]

*A **median** is a segment that connects any vertex of a triangle to the midpoint of its opposite side.*

Centroid	
Description	**Figure**
The medians of a triangle intersect at a point called the centroid that is two thirds of the distance from each vertex to the midpoint of the opposite side.	

Examples



a. In ΔABC, Q is the centroid and $BE = 9$. Find BQ and QE.

$BQ = \frac{2}{3} BE$ $\quad QE = \frac{1}{3} BE$

$BQ = \frac{2}{3}(9)$ $\quad QE = \frac{1}{3}(9)$

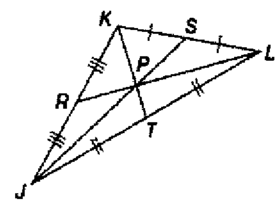
$BQ = 6$ $\quad QE = 3$

b. In ΔABC, $FC = 15$. Find FQ and QC.

$FQ = \frac{1}{3} FC$ $\quad QC = \frac{2}{3} FC$

$FQ = \frac{1}{3}(15)$ $\quad QC = \frac{2}{3}(15)$

$FQ = 5$ $\quad QC = 10$



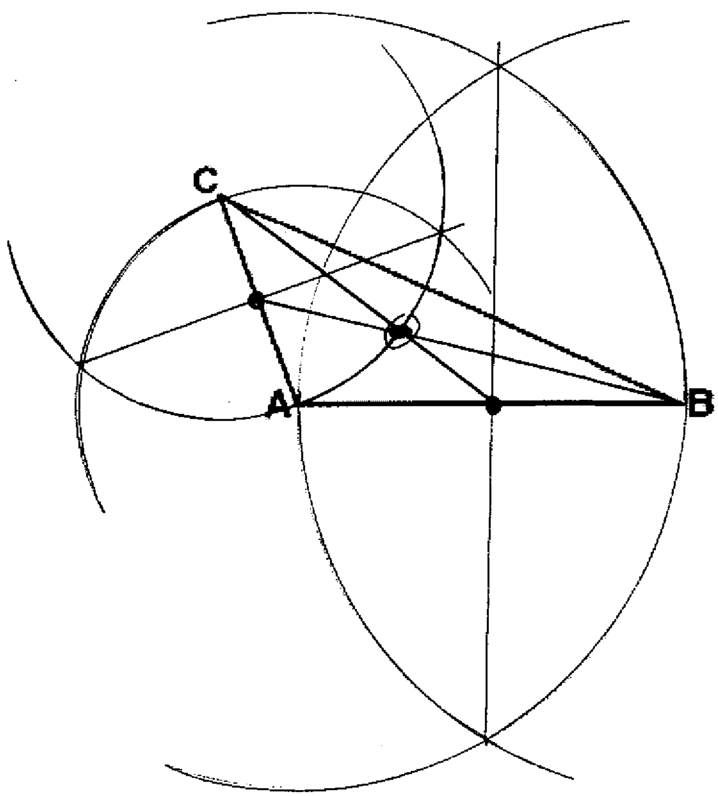
c. In ΔJKL, $PT = 2$. Find KP.

$KP = 2(PT)$

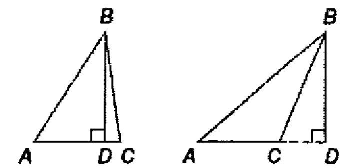
$KP = 2(2)$

$KP = 4$

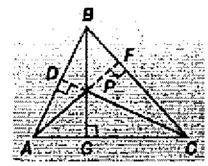
Given ΔABC, locate (by construction) the centroid of the triangle.



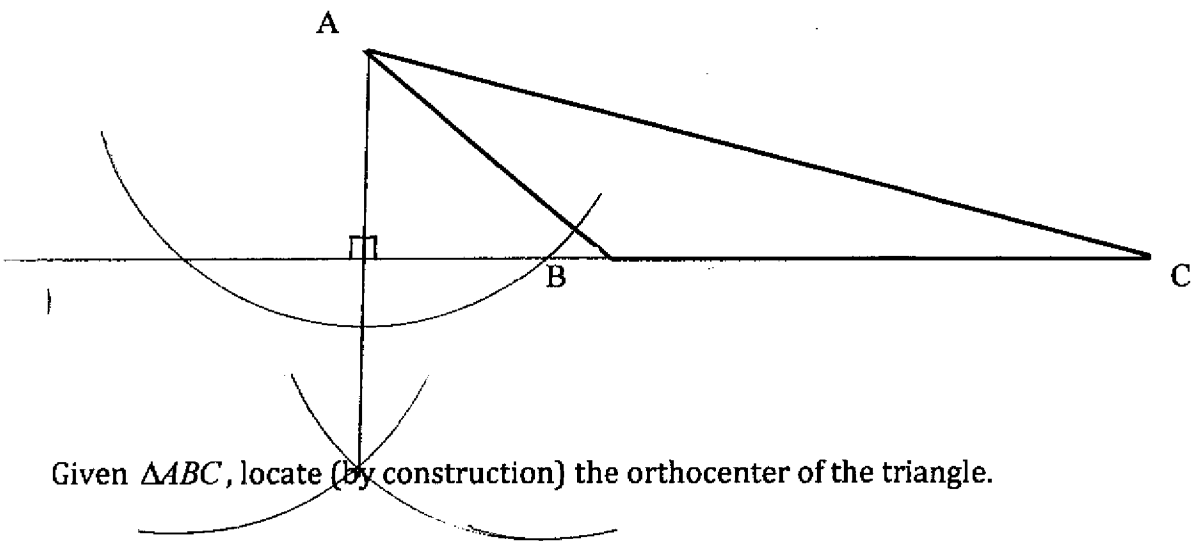
*An **altitude** is a segment drawn from any vertex of a triangle perpendicular to its opposite side.*



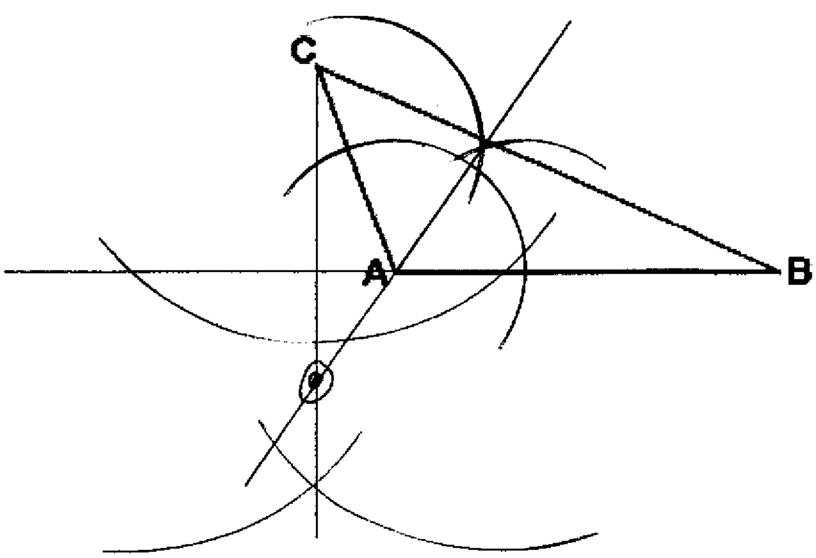
$\overline{BD}$ is an altitude from B to $\overline{AC}$.

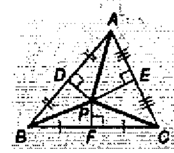
[illegible]	
Description	**Figure**
The lines containing the altitudes of a triangle are concurrent, intersecting at a point called the orthocenter	

Construct the **altitude** from vertex A to side $\overline{BC}$ in triangle ABC.

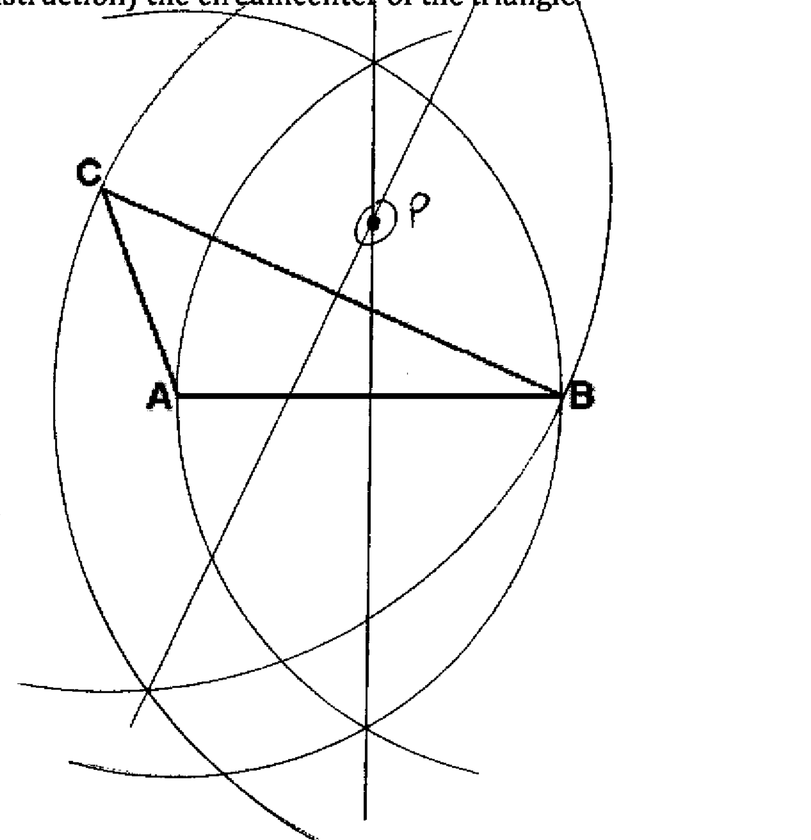


Given ΔABC, locate (by construction) the orthocenter of the triangle.



Circumcenter	
Description	Figure
The perpendicular bisectors of a triangle intersect at the circumcenter, which is equidistant from the vertices of the triangle.	

Given ΔABC, locate (by construction) the circumcenter of the triangle.



a. The circumcenter of ΔABC is at P. AP = x – y, BP = 4y, and CP = 4. Find x and y.

P is equidistant from the vertices of ΔABC (AP = BP = CP)

$4y = 4$
$y = 1$

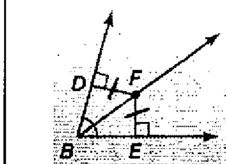
$x - y = 4$
$x - 1 = 4$
$x = 5$

b. The circumcenter of ΔABC is at P. AP = 3x – y, BP = x + y, and CP = 4. Find x and y.

$3x - y = x + 4$
$2x - y = 4$
$2x - 4 = y$

$x + y = 4$
$x + 2x - 4 = 4$
$3x = 8$
$x = \frac{8}{3}$

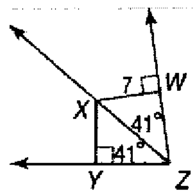
$\frac{8}{3} + y = 4$
$y = 4 - \frac{8}{3}$
$y = \frac{4}{3}$

Angle Bisectors		
Theorem		Figure
Angle Bisector Theorem	If a point is on the angle bisector of an angle, then it is equidistant from the sides of the angle.	

Examples

Find each measure.

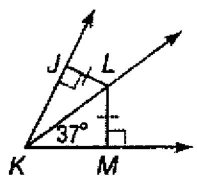
a. *XY*



$XY = XW$

$XY = 7$

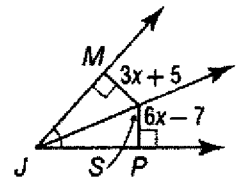
b. $m\angle JKL$



$m\angle JKL = m\angle MKL$

$m\angle JKL = 37°$

c. *SP*



$MS = SP$

$3x + 5 = 6x - 7$

$12 = 3x$

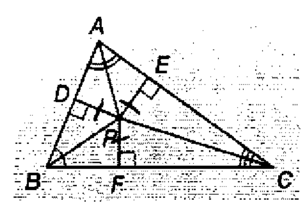
$x = 4$

$SP = 6(4) - 7$

$SP = 24 - 7$

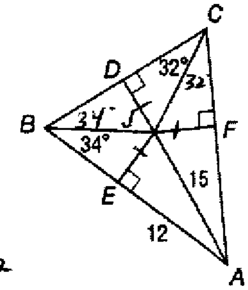
$SP = 17$

Similar to perpendicular bisectors, since a triangle has three angles, it also has three angle bisectors. The angle bisectors of a triangle are concurrent, and their point of concurrency is called the ***incenter*** *of the triangle.*

Incenter	
Description	Figure
The angle bisectors of a triangle intersect at the incenter, which is equidistant from the sides of the triangle.	

Examples

Find each measure if J is the incenter of $\triangle ABC$.



a. *JF*

$JF = JE$

$(JE)^2 + 12^2 = 15^2$

$(JE)^2 + 144 = 225$

$(JE)^2 = 81$

$JE = \sqrt{81}$

$JE = 9$

$JF = 9$

b. $\angle ABC$

$m\angle ABC = 2m\angle EBJ$

$m\angle ABC = 2(34)$

$m\angle ABC = 68°$

c. $\angle JAC$

$2(34) + 2(32) =$

$68 + 64 =$

132

$180 - 132 = 48°$

$m\angle JAC = 48/2$

$m\angle JAC = 24°$

Given ΔABC, locate (by construction) the incenter of the triangle.