Geometry R CC – Review Sheet
Unit 11 – Three Dimensional Geometry

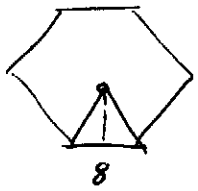
Name _____
Period_____ Date_____

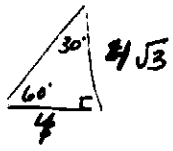
1) A regular hexagon is divided into 6 congruent triangles. If the perimeter of the hexagon is 48 centimeters, what is the height of each triangle?

A) 4 *cm* B) 8 *cm*

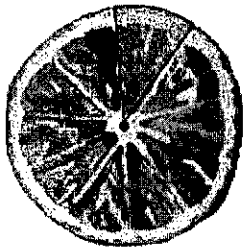
C) $4\sqrt{3}$ *cm* D) $8\sqrt{3}$ *cm*

$\frac{48}{6} = 8$





2) The diameter of the orange slice shown is 9 centimeters. If each of the orange's 10 sections are congruent, find the area of 8 sections of the orange to the nearest tenth.



$A_o = \pi r^2$

$A_o = \pi (4.5)^2$

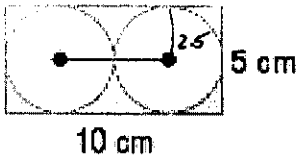
$A_o = 20.25\pi \text{ cm}^2$

$A = \frac{8}{10}(20.25\pi)$

$= 50.893800999$

$\approx 50.9 \text{ cm}^2$

3) Find the area of the shaded region to the nearest hundredth.



$A_o = \pi r^2$

$= \pi (2.5)^2$

$= 6.25\pi$

$A_{\square} = lw$

$= 10(5)$

$= 50 \text{ cm}^2$

$A_{2o} = 2(6.25\pi)$

$= 12.5\pi \text{ cm}^2$

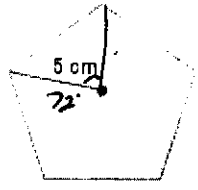
$A = 50 - 12.5\pi$

$A = 10.73009183$

$A \approx 10.73 \text{ cm}^2$

4) Find the area of the polygon to the nearest tenth.

a)



$\frac{360}{5} = 72°$

$K = \frac{1}{2} ab \sin C$

$= \frac{1}{2}(5)(5)\sin 72°$

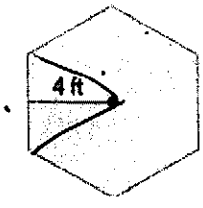
$= 11.88820645$

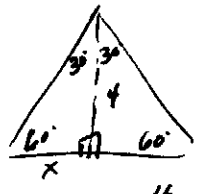
$A = 5(11.88820645)$

$A = 59.44103227$

$A = 59.4 \text{ cm}^2$

b)





$\tan 60° = \frac{4}{x}$

$x = \frac{4}{\tan 60°}$

$A_{\triangle} = \frac{1}{2}bh$

$= \frac{1}{2}\left(\frac{8}{\tan 60°}\right)(4)$

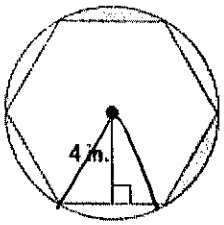
$= \frac{16}{\tan 60°}$

$A = 6\left(\frac{16}{\tan 60°}\right)$

$= 55.4256$

$\approx 55.4 \text{ ft}^2$

5) Find the area of the shaded region. Round to the nearest tenth.



$\sin 60° = \frac{4}{x}$

$x = \frac{4}{\sin 60°}$

radius

$A_\Delta = \frac{1}{2} ab \sin C$

$= \frac{1}{2}\left(\frac{4}{\sin 60°}\right)\left(\frac{4}{\sin 60°}\right) \sin 60°$

$= \frac{8}{\sin 60°}$

$A_{Hex} = 6\left(\frac{8}{\sin 60°}\right)$

$= \frac{48}{\sin 60°}$

$A_0 = \pi r^2$

$= \pi\left(\frac{4}{\sin 60°}\right)^2$

$A_1 = \pi\left(\frac{4}{\sin 60°}\right)^2 - \frac{48}{\sin 60°}$

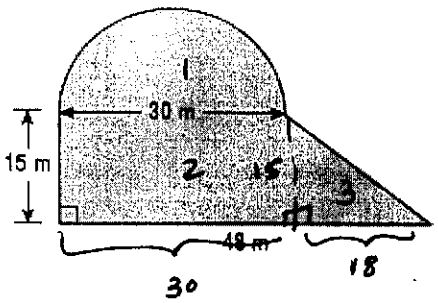
$A_1 = 11.59501743$

$A = \frac{1}{2}(11.59501743)$

$A = 5.797508717$

$\approx 5.8 \text{ in}^2$

6) Find the area of the figure.



$A_0 = \pi r^2$

$A_1 = \frac{1}{2}\pi(15)^2$

$A_1 = 112.5 \text{ m}^2$

$A_2 = lw$

$= 15(30)$

$= 450 \text{ m}^2$

$A_3 = \frac{1}{2}bh$

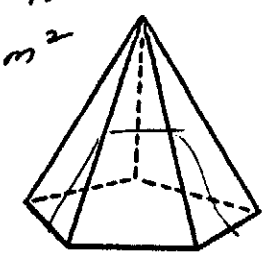
$= \frac{1}{2}(18)(15)$

$= 135$

$A = 112.5 + 450 + 135$

$A = 697.5 \text{ m}^2$

7) Use the figure at the right to answer parts a – c.



a) Describe the intersection of the pyramid and a vertical plane that passes through the vertex of the pyramid.

an isosales Δ

b) Describe the intersection of the pyramid and a vertical plane that passes through the pyramid but not its vertex.

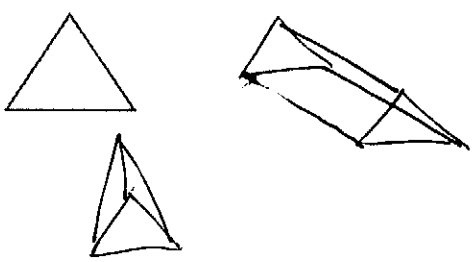
A quadrilateral, triangle, or trapezoid

c) Describe the intersection of the pyramid and a horizontal plane that passes through the pyramid but not its vertex.

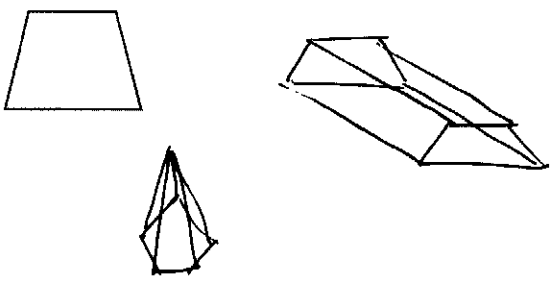
a pentagon

8) For each of the following cross-sections, sketch the figure from which the cross-sections was taken.

a)



b)



9) Describe the cross-section of each of the following objects.

a) right circular cone cut by a plane through the vertex and perpendicular to the base

an isosceles $\triangle$

b) square pyramid cut by a plane through the vertex and perpendicular to the base

an isosceles $\triangle$

c) square pyramid cut by a vertical plane that is parallel to an edge of the base but not passing through the vertex

an isosceles trapezoid

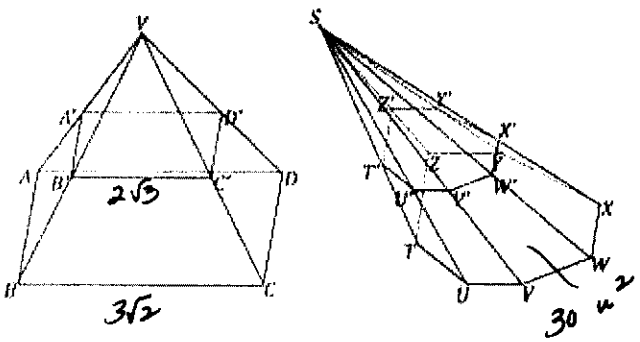
d) triangular prism cut by a plane parallel to a base

a triangle equal in area to the base

e) triangular prism cut by a plane parallel to a face

rectangle

10) The following pyramids have equal altitudes, and both bases are equal in area and coplanar. Both pyramids' cross-sections are coplanar. If $BC = 3\sqrt{2}$ and $B'C' = 2\sqrt{3}$, and the area of $TUVWXYZ$ is 30 $units^2$, what is the area of cross-section $A'B'C'D'$?



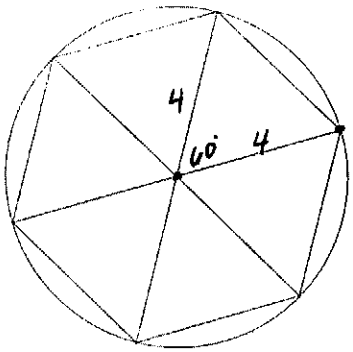
$$\left(\frac{2\sqrt{3}}{3\sqrt{2}}\right)^2 = \frac{A}{30}$$

$$\frac{12}{18} = \frac{A}{30}$$

$$360 = 18A$$

$$A = 20 \ u^2$$

11) a) Approximate the area of a disk of radius 4 using an inscribed regular hexagon. What is the percent error of the approximation?



$A_\Delta = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(4)(4)\sin 60°$

$= 8\sin 60°$

$A_{hex} = 6(8\sin 60°)$

$= 48\sin 60°\ u^2$

$A_\odot = \pi r^2$

$= \pi(4^2)$

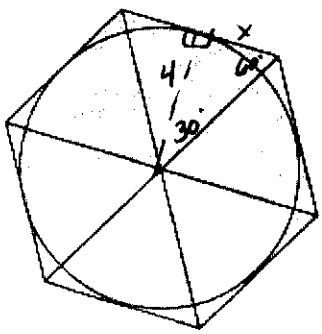
$= 16\pi\ u^2$

$\%\ error = \frac{|0-a|}{a} \cdot 100\%$

$= \frac{|48\sin 60° - 16\pi|}{16\pi} \cdot 100\%$

$= 17.30066569\ \%$

b) Approximate the area of a circle of radius 4 using a circumscribed regular hexagon. What is the percent error of the approximation?



$\tan 60° = \frac{4}{x}$

$x = \frac{4}{\tan 60°}$

$A_\Delta = \frac{1}{2}bh$

$= \frac{1}{2}\left(\frac{8}{\tan 60°}\right)(4)$

$= \frac{16}{\tan 60°}$

$A_{hex} = 6\left(\frac{16}{\tan 60°}\right)$

$= \frac{96}{\tan 60°}$

$A_\odot = 16\pi$

$\%\ error = \frac{|0-a|}{a} \cdot 100\%$

$= \frac{\left|\frac{96}{\tan 60°} - 16\pi\right|}{16\pi} \cdot 100\%$

$= 10.26577908\%$

c) Find the average of the approximations for the area of a circle of radius 4 using inscribed and circumscribed regular hexagons. What is the percent error of the average approximation?

$A = \frac{48\sin 60° + \frac{96}{\tan 60°}}{2}$

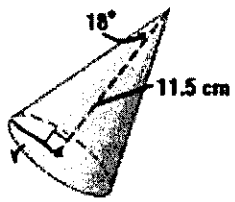
$= 48.49742261\ u^2$

$\%\ error = \frac{|48.49742261 - 16\pi|}{16\pi} \cdot 100\%$

$= 3.517443301\%$

For #12 – 27, find the volume of each solid to the nearest tenth.

12)



$\tan 18^\circ = \frac{r}{11.5}$

$r = 11.5 \tan 18^\circ$

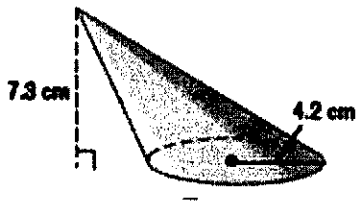
$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (11.5 \tan 18^\circ)^2 (11.5)$

$V = 168.1412285$

$V \approx 168.1 \text{ cm}^3$

13)



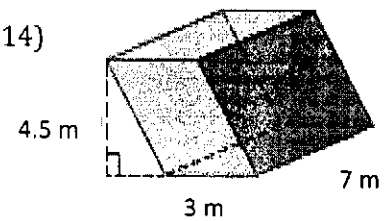
$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi (4.2)^2 (7.3)$

$V = 134.8497231$

$V \approx 134.8 \text{ cm}^3$

14)



$V = lwh$

$V = 3(7)(4.5)$

$V = 94.5 \text{ m}^3$

15)

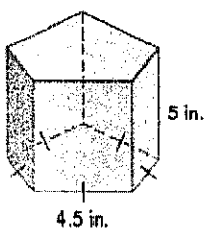
$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

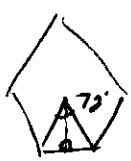
$V = \frac{4}{6}\pi (5^3)$

$V = 261.7993878$

$V \approx 261.8 \text{ ft}^3$

16)





$\frac{360}{5} = 72^\circ$

$\tan 36^\circ = \frac{2.25}{h}$

$h = \frac{2.25}{\tan 36^\circ}$

$A_\Delta = \frac{1}{2}bh$

$= \frac{1}{2}(4.5)\left(\frac{2.25}{\tan 36^\circ}\right)$

$= 6.967933472$

$B = 5(6.967933472)$

$= 34.83966736$

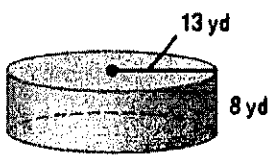
$V = Bh$

$= 34.83966736(5)$

$= 174.1983368$

$\approx 174.2 \text{ in}^3$

17)



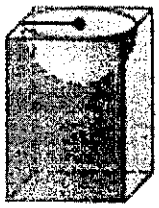
$V = \pi r^2 h$

$= \pi (13^2)(8)$

$= 1352\pi$

$= 4247.433268$

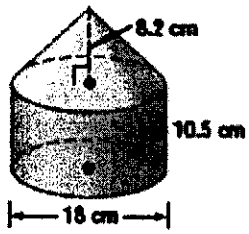
$\approx 4,247.4 \text{ yd}^3$

18) 

$V_R = lwh$
$= 18(18)(20)$
$= 6480\ cm^3$

$V_H = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$
$= \frac{4}{6}\pi(9^3)$
$= 486\pi\ cm^3$

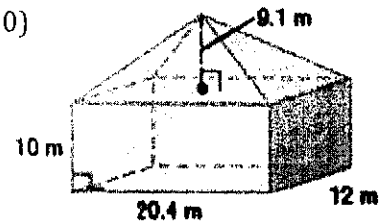
$V = 6480 - 486\pi$
$= 4953.18597$
$\approx 4953.2\ cm^3$

19) 

$V_{cyl.} = \pi r^2 h$
$= \pi(9^2)(10.5)$
$= 850.5\pi$

$V_{cone} = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(9^2)(8.2)$
$= 221.4\pi$

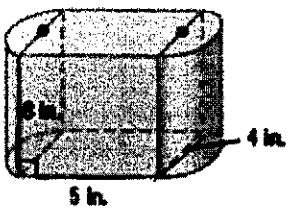
$V = 850.5\pi + 221.4\pi$
$V = 3367.473165$
$V \approx 3367.5\ cm^3$

20) 

$V_p = lwh$
$= 20.4(12)(10)$
$= 2448\ m^3$

$V_{pyr.} = \frac{1}{3}lwH$
$= \frac{1}{3}(20.4)(12)(9.1)$
$= 742.56$

$V = 2448 + 742.56$
$V = 3190.56\ m^3$
$V \approx 3190.6\ m^3$

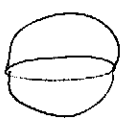
21) 

$V_{cyl} = \pi r^2 h$
$= \pi(2^2)(8)$
$= 32\pi\ in^3$

$V_{prism} = lwh$
$= 5(4)(8)$
$= 160\ in^3$

$V = 32\pi + 160$
$V = 260.5309649$
$V \approx 260.5\ in^3$

22) A sphere whose great circle has an area of $49\pi m^2$



$A = \pi r^2$
$49\pi = \pi r^2$
$r = 7\ m$

$V = \frac{4}{3}\pi r^3$
$V = \frac{4}{3}\pi(7^3)$
$V = 1436.755504$
$V \approx 1436.8\ m^3$

23) A hemisphere whose great circle has a circumference of 18 feet

$C = 2\pi r$

$\frac{18}{2\pi} = \frac{2\pi r}{2\pi}$

$r = \frac{9}{\pi}$ ft

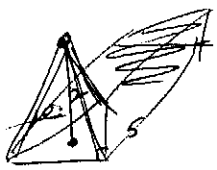
$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \frac{2}{3}\pi\left(\frac{9}{\pi}\right)^3$

$= 49.24209525$

≈ 49.2 ft^3

24) A triangular pyramid with a height of 4.8 cm and a right triangular base with a leg 5 cm and a hypotenuse of 10.2 cm



$V = \frac{1}{3}BH$

$V = \frac{1}{3}(22.22611077)(4.8)$

$V = 35.56177723$

$V \approx 35.6$ cm^3

$B = \frac{1}{2}bh$

$B = \frac{1}{2}(\sqrt{79.04})(5)$

$B = 22.22611077$ cm^2

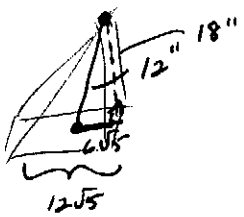
$5^2 + b^2 = 10.2^2$

$25 + b^2 = 104.04$

$b^2 = 79.04$

$b = \sqrt{79.04}$

25) A square pyramid with an altitude of 12 inches and a slant height of 18 inches.



$a^2 + 12^2 = 18^2$

$a^2 + 144 = 324$

$a^2 = 180$

$a = \sqrt{180}$

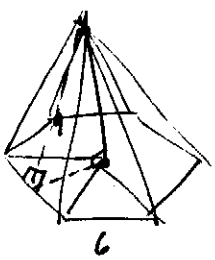
$a = 6\sqrt{5}$

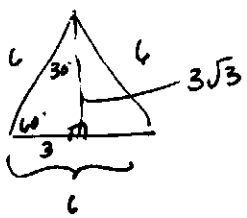
$V = \frac{1}{3}s^2h$

$V = \frac{1}{3}(12\sqrt{5})^2(12)$

$V = 2880$ in^3

26) A hexagonal pyramid with a base edge of 6 mm and a slant height of 9 mm.





$A_\Delta = \frac{1}{2}(6)(3\sqrt{3})$

$= 9\sqrt{3}$

$B = 6(9\sqrt{3})$

$= 54\sqrt{3}$ mm^2

$(3\sqrt{3})^2 + h^2 = 9^2$

$27 + h^2 = 81$

$\sqrt{h^2} = \sqrt{54}$

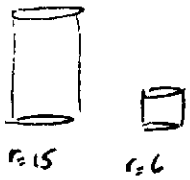
$h = 3\sqrt{6}$

$V = \frac{1}{3}(54\sqrt{3})(3\sqrt{6})$

$V = 54\sqrt{18}$

$V = 162\sqrt{2}$ mm^3

27) Two similar cylinders have radii of 15 inches and 6 inches. What is the ratio of the volume of the small cylinder to the volume of the large cylinder?



$\left(\frac{6}{15}\right)^3 = \frac{216}{3375} = \frac{8}{125}$

28) A rectangular prism has a base with sides of x and $x + 3$ and height $2x$. Find the volume of the prism in terms of x.

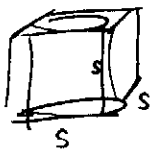
$V = lwh$

$V = x(x+3)(2x)$

$V = 2x^2(x+3)$

$V = 2x^3 + 6x^2$

29) A cube with sides of S has a drill pass perpendicularly through it, removing the shape of a cylinder, which has a radius of R. Write an expression that best estimates the remaining volume of the cube, once the drill has passed through?



$V_{cube} = S^3$ $V_{cylinder} = \pi r^2 h = \pi R^2 S$

$V = S^3 - \pi R^2 S$

30) Prism A has a height of 5.0 and a mass of 1000. Similar prism B has a height of 7.0 and a mass of 2744. Which prism has the greater density?

$d = \frac{m}{V}$ $r = \frac{5}{7}$ $r^3 = \left(\frac{5}{7}\right)^3 = \frac{125}{343}$

$d_A = \frac{1000}{125}$ $d_B = \frac{2744}{343}$

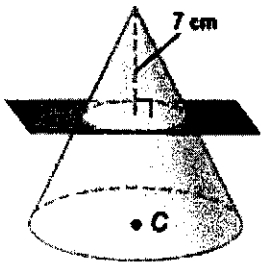
$d_A = 8$ $d_B = 8$

They have the same densities.

31) You want to know if the volumes of two stacks of playing cards will be equal. What has to be true about the two stacks for this to be true?

By Cavalieri's Principle two prisms will have equal volumes if their heights are equal and the area at any cross section are equal. Since the area of each playing card is the same, the heights would need to be equal.

32) Plane P is parallel to the base of cone C, and the volume of the cone above the plane is $\frac{1}{8}$ of the volume of the cone below the plane. Find the height of the cone C.



$\sqrt[3]{r^3} = \sqrt[3]{\frac{1}{8}}$

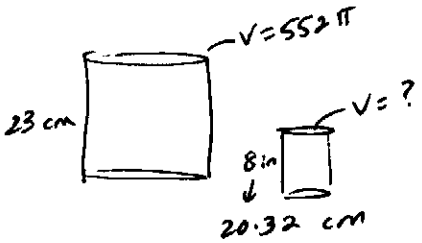
$r = \frac{1}{2}$

$\frac{1}{2} = \frac{7}{h}$

$14 = h$ cm

33) Two cylinders are similar. The height of the first cylinder is 23 *cm* and the height of the other cylinder is 8 *in*. If the volume of the first cylinder is $552\pi\ cm^3$, what is the volume of the other prism?

Use 2.54 *cm* = 1 *in*.



$\frac{8\text{ in}}{1} \cdot \frac{2.54\text{ cm}}{1\text{ in}} = 20.32\text{ cm}$

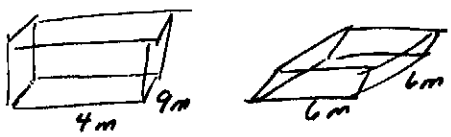
$\left(\frac{23}{20.32}\right)^3 = \frac{552\pi}{V}$

$\frac{12,167}{8390.176768} = \frac{552\pi}{V}$

$\frac{12167V}{12167} = \frac{14549901.77}{12167}$

$V = 1195.849574$ cm^3

34) A right rectangular prism with a base of 4 meters by 9 meters has the same volume as an oblique rectangular prism with a base of 6 meters by 6 meters. What can you conclude about the heights of the two prisms and why?



The heights must be the same by Cavalieri's Principle since their bases both have the same area and all their cross sections are = since they are prisms.

35) Two suitcases are similar rectangular prisms. The smaller suitcase is 68 cm long, 47 cm wide, and 27 cm deep. The larger suitcase is 85 cm long.

a) What is the scale factor of the prisms?

$r = \frac{68}{85} = \frac{4}{5}$

b) What is the volume of the larger suitcase to the nearest tenth?

$V_{smaller} = lwh$

$= 68(47)(27)$

$= 86,292\ cm^3$

$\left(\frac{4}{5}\right)^3 = \frac{86,292}{V}$

$\frac{64}{125} = \frac{86,292}{V}$

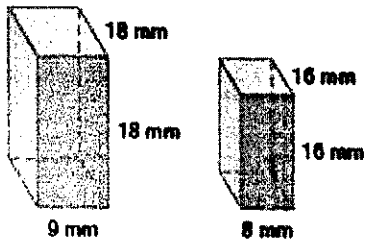
$V = 168,539.0625$

$168,539.1\ cm^3$

36) Determine whether each pair of solids is similar, congruent, or neither.

If the solids are similar, state the scale factor.

a)

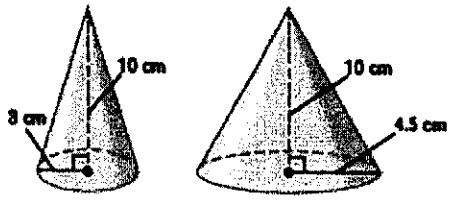


$\frac{9}{8} \neq \frac{18}{16} = \frac{18}{16} = \frac{9}{8}$

$r = \frac{8}{9}$

yes the prisms are similar

b)

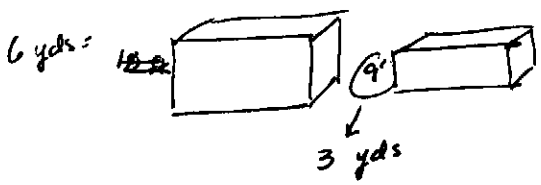


$\frac{10}{10} \neq \frac{8}{4.5}$

$45 \neq 30$

no the cones are not similar

37) Two rectangular prisms are similar. The height of the first prism is 6 *yards* and the height of the other is 9 *feet*. If the volume of the first prism is 810 *cubic yards*, what is the volume of the other prism?



$\left(\frac{3}{6}\right)^3 = \frac{V}{810}$ $\frac{27}{216} = \frac{V}{810}$ $21,870 = 216\,V$

$V = 101.25\ yds^3$

~~$5940 = 5832\,V$~~

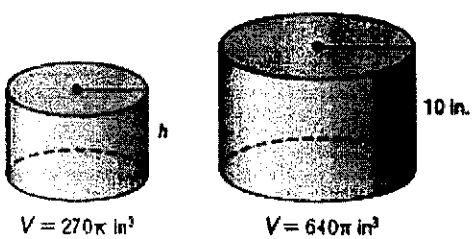
~~$V = 101.25$~~

38) Two spheres have surface areas of 100π square centimeters and 16π square centimeters. What is the ratio of the volume of the large sphere to the volume of the small sphere?

$r^2 = \frac{100\pi}{16\pi}$ $r^3 = \frac{10^3}{4^3}$ $r^3 = \frac{1000}{64}$ $\frac{125}{8}$

$r = \frac{10}{4}$

39) The containers below are similar cylinders. Find the height, h, of the smaller container.



$\left(\frac{10}{h}\right)^3 = \frac{640\pi}{270\pi}$

$\frac{1000}{h^3} = \frac{640}{270}$

$\frac{640\,h^3}{640} = \frac{270{,}000}{640}$

$\sqrt[3]{h^3} = \sqrt[3]{\frac{27{,}000}{64}}$

$h = \frac{30}{4}$

$h = 7.5$ inches

40) A cylindrical can has a volume of 363 cm^3. The diameter of the can is 9 cm. What is the height of the can?

$V = 363\ cm^3$

$V = \pi r^2 h$

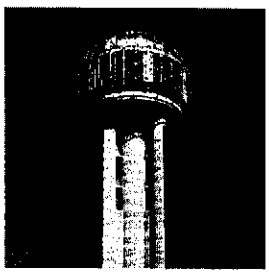
$363 = \pi (4.5^2) h$

$\frac{363}{20.25\pi} = \frac{20.25\pi h}{20.25\pi}$

$h = 5.705999441$

$5.7\ cm$

41) The Reunion Tower in Dallas, Texas, is topped by a spherical dome that has a surface area of approximately 13,924 π square feet. What is the volume of the dome to the nearest tenth?



$SA = 4\pi r^2$

$\frac{13924\pi}{4\pi} = \frac{4\pi r^2}{4\pi}$

$\sqrt{3481} = \sqrt{r^2}$

$r = 59$

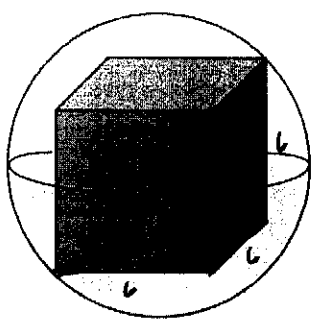
$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi (59)^3$

$V = 860,289.5435$

$V \approx 860,289.5\ ft^3$

42) A cube has a volume of 216 in^3. Find the volume of a sphere that is circumscribed about the cube. Round to the nearest tenth.



$V = s^3$

$\sqrt[3]{216} = \sqrt[3]{s^3}$

$s = 6$

$6^2 + 6^2 = x^2$

$72 = x^2$

$72 + 6^2 = y^2$

$72 + 36 = y^2$

$108 = y^2$

$y = 6\sqrt{3}$

↑ diameter

$r = 3\sqrt{3}$

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (3\sqrt{3})(3\sqrt{3})(3\sqrt{3})$

$= \frac{4}{3}\pi (27\sqrt{27})$

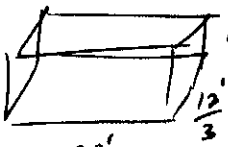
$= \frac{4}{3}\pi (27)(3\sqrt{3})$

$= 587.670994$

$\approx 587.7\ in^3$

43) Mr. Mattina is planning to remove an old patio and install a new rectangular concrete patio that is 20 feet long, 12 feet wide, and 4 inches thick. One contractor bid $2225 for the entire project.

A second contractor bid $500 per cubic yard for the new patio and $700 for the removal of the old patio. Which is the less expensive option? Explain.



$4'' = \frac{4}{12}' = \frac{1}{3}'$

$\frac{1}{9}$ yd

$\frac{12'}{3} = 4$ yds

$\frac{20'}{3}$ yds

$V = lwh$

$V = \left(\frac{20}{3}\right)(4)\left(\frac{1}{9}\right)$

$V = \frac{80}{27}\ yd^3$

Contractor 2

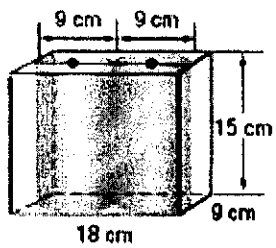
$\$500/yd^3 \left(\frac{80}{27}\ yd^3\right) = 1481.481481$

$+ \ 700$

$\$2181.481481$

The second contractor is less expensive since $\$2,181.48 < \$2,225$.

44) A box 18 cm by 9 cm by 15 cm is being used to ship two cylindrical candles. Each candle has a diameter of 9 cm and a height of 15 cm, as shown in the diagram. What is the volume of the empty space in the box?



$V_{cyl.} = \pi r^2 h$

$V_{cyl} = \pi (4.5)^2 (15)$

$= 303.75\pi\ cm^3$

$V_{Box} = lwh$

$= 18(9)(15)$

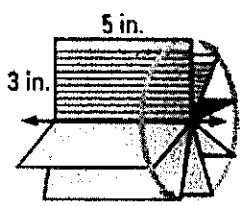
$= 2430\ cm^3$

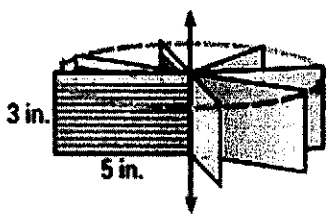
$V = 2430 - 2(303.75\pi)$

$V = 521.4824629$

$\approx 521.4824629\ cm^3$

45) Suppose that a 3 inch by 5 inch index card is rotated around a horizontal line and a vertical line to produce two different solids, as shown. Which solid has a greater volume? Explain your reasoning.





$V_1 = \pi r^2 h$

$= \pi (3^2) 5$

$= 45\pi$

$V_2 = \pi r^2 h$

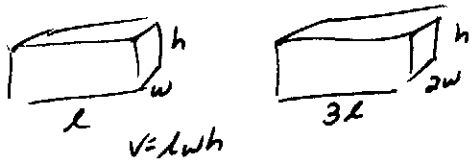
$= \pi (5^2)(3)$

$= 75\pi$

The second solid has a greater volume since $75\pi > 45\pi$.

46) The length of a rectangular prism is tripled, its width is doubled, and its height remains the same, what is the volume of the new rectangular prism?

A) double the original volume
B) nine times the original volume
C) triple the original volume
D) six times the original volume

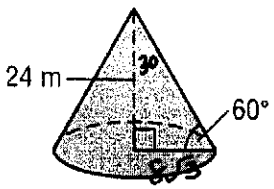


$V = \ell w h$

$V = (3\ell)(2w)h$
$= 6\ell wh$

47) Find the exact volume of the cone to the right:

A) $V = 48\pi$
B) $V = 64\pi$
C) $V = 144\pi$
D) $V = 192\pi$



$\ell = s\sqrt{3}$

$\frac{24}{\sqrt{3}} = \frac{s\sqrt{3}}{\sqrt{3}}$

$s = \frac{24}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3}$

$V = \frac{1}{3}\pi r^2 h$

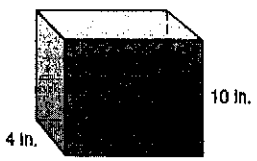
$V = \frac{1}{3}\pi (8\sqrt{3})^2 (24)$

$= \frac{1}{3}\pi (64 \cdot 3)(24$

$V = 1536\pi \ m^3$

48) Prisms A and B have the same length and width, but different heights. If the volume of Prism B is 150 cubic inches greater than the volume of Prism A, what is the length of each prism?

Prism A



Prism B

$V_A = Bh$

$V_B = Bh$

$10B = 7B + 150$

$3B = 150$

$B = 50$

$B = \ell w$

$\frac{50}{4} = \frac{\ell(4)}{4}$

$\ell = 12.5$ in.

49) Two tents are in the shape of hemispheres, with circular floors. The ratio of their floor areas is 9:12.25. If the diameter of the smaller tent is 6 feet, what is the volume of the larger tent?

$\sqrt{r^2} = \sqrt{\frac{9}{12.25}}$

$r = \frac{3}{3.5}$

$\frac{3}{3.5} = \frac{6}{d}$

$3d = 21$

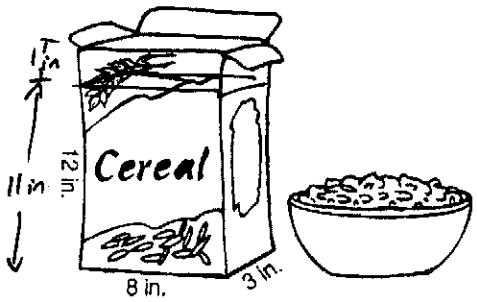
$d = 7$

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \frac{2}{3}\pi (3.5^3)$

$= \frac{343\pi}{12}$

50) A bulk cereal producer wants to fill a jumbo size box with its brand of rice flakes. They decide to fill the box 1 inch from the top instead of filling it completely. How many cubic inches of rice flakes will the producer save by not filling the box to the top?



$V = lwh$

$V = 8(3)(1)$

$V = 24\ in^3$

51) Each stack of memo papers shown contains 500 sheets of paper.

a) Explain why the stacks have the same volume.

Cavalieri's Principle

Since the two prisms have the same height and the areas at every cross section are equal.

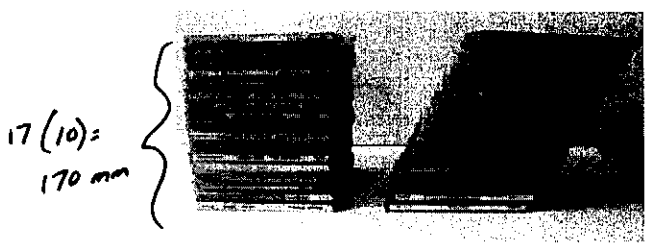
b) Calculate the volume of the stack if each sheet of paper is 3 inches by 3 inches by 0.01 inches.

$V = lwh$

$V = (3)(3)(0.01 \cdot 500)$

$V = 4.5\ in^3$

52) Find the volume of both prisms and explain how Cavalieri's Principle applies. A CD case measures 142mm by 125mm by 10mm.



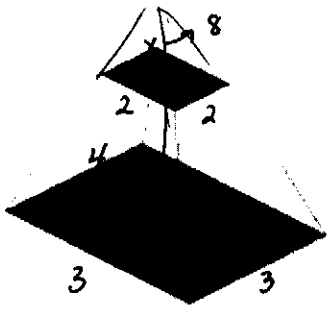
The volumes are equal since the height of the two prisms are the same and the area at every cross section is equal. (Cavalieri's Principle)

$V = lwh$

$V = 142(125)(170)$

$V = 3017500\ mm^3$

53) The frustum of a pyramid is formed by cutting off the top part by a plane parallel to the base. The base of the pyramid and the cross-section where the cut is made are called the bases of the frustum. The distance between the planes containing the bases is called the height of the frustum. Find the volume of the frustum if the bases are squares of edge lengths 2 and 3, and the height of the frustum is 4.



$V = \frac{1}{3}Bh$

$V = \frac{1}{3}s^2h$

$V = \frac{1}{3}(3^2)(12)$

$V = 36$ ← entire pyramid

$\frac{x}{x+4} = \frac{2}{3}$

$3x = 2(x+4)$

$3x = 2x + 8$

$x = 8$

$\left(\frac{2}{3}\right)^3 = \frac{V}{36}$

$\frac{8}{27} = \frac{V}{36}$

$288 = 27V$

$V = \frac{32}{3}\ u^3$ ← smaller pyramid

$V = 36 - \frac{32}{3}$

$= \frac{76}{3}\ u^3$ ← volume of frustum

54) Use the diagram below to answer the questions that follow.

a) Determine the volume of the cone shown below. Give an exact answer.

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(4^2)(11)$

$= \frac{176\pi}{3}\ u^3$

$11^2 + r^2 = \sqrt{137}^2$

$121 + r^2 = 137$

$r^2 = 16$

$r = 4$

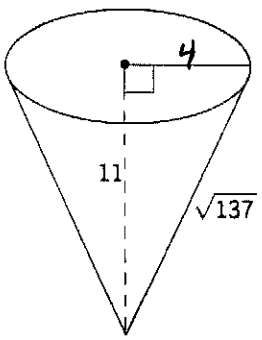
b) Find the dimensions of a cone that is similar to the one given above. Explain how you found your answers.

$r = 8$

$h = 22$

I multiplied the radius and height by 2.

c) Calculate the volume of the cone that you described in part (b) using two different methods (using normal volume formulas and using the scaling principle for volume)



$\left(\frac{4}{8}\right)^3 = \frac{\frac{176\pi}{3}}{V}$

$\frac{1}{8} = \frac{\frac{176\pi}{3}}{V}$

$64V = \frac{11264\pi}{3}$

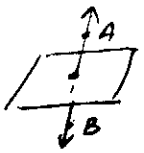
$V = \frac{704\pi}{3}$

$V = \frac{1408\pi}{3}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(8^2)(22)$

$= \frac{1408\pi}{3}\ u^3$

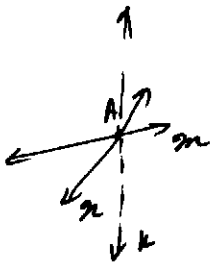


55) If line $\overline{AB}$ is perpendicular to a plane, how many planes containing $\overline{AB}$ can be drawn parallel to the plane?

(A) none (B) one (C) two (D) infinitely many

56) Lines *m* and *n* intersect at point *A*. Line *k* is perpendicular to both lines *m* and *n* at point *A*.

Which statement must be <u>true</u>?



(A) Lines *m*, *n*, and *k* are in the same plane.

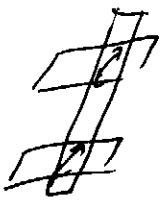
(B) Lines *m* and *n* are in two different planes.

(C) Lines *m* and *n* are perpendicular to each other.

(D) Line *k* is perpendicular to the plane containing lines *m* and *n*.

57) In three-dimensional space, two planes are parallel and a third plane intersects both of the parallel planes.

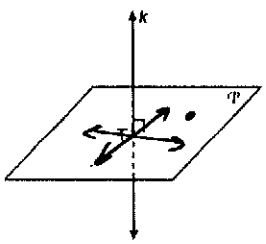
The intersection of the planes is a



(A) plane (B) point

(C) pair of parallel lines (D) pair of intersecting lines

58) In the diagram at the right, line *k* is perpendicular to plane *P* at point *T*. Which statement is <u>true</u>?



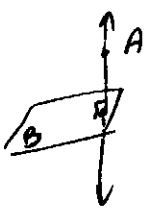
(A) Any point in plane *P* also will be on line *k*.

(B) Only one line in plane *P* will intersect line *k*.

(C) All planes that intersect plane *P* will pass through *T*.

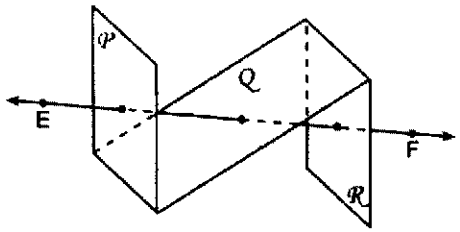
(D) Any plane containing line *k* is perpendicular to plane *P*.

59) Point A is not contained in plane B. How many lines can be drawn through point A that will be perpendicular to plane B?



(A) one (B) zero (C) two (D) infinitely many

60) As shown in the diagram at the right, $\overleftrightarrow{EF}$ intersects planes P, Q, and R. If $\overleftrightarrow{EF}$ is perpendicular to planes P and R, which statement must be true?



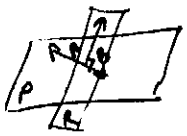
(A) Plane P is perpendicular to plane Q.

(B) Plane R is perpendicular to plane P.

(C) Plane P is parallel to plane Q.

(D) Plane R is parallel to plane P.

61) If $\overleftrightarrow{AB}$ is contained in plane P, and $\overleftrightarrow{AB}$ is perpendicular to plane R, which statement is true?



(A) $\overleftrightarrow{AB}$ is parallel to plane R.

(B) Plane P is parallel to plane R.

(C) $\overleftrightarrow{AB}$ is perpendicular to plane P.

(D) Plane P is perpendicular to plane R.

62) Which of the following statements is false?

(1) If two lines are parallel, then they must be coplanar.

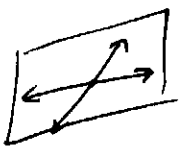
(2) Every triangle is a plane figure.

(3) If two lines are parallel to a third line, then all 3 lines are coplanar.

(4) Two perpendicular lines must be coplanar.

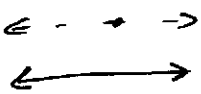
63) If two lines have exactly one point in common, how many planes contain those lines?

(1) none (2) one (3) two (4) infinitely many



64) How many lines are parallel to a given line through a given point not on that line?

(1) none (2) one (3) two (4) infinitely many



65) Through a point outside a given plane, how many planes can be drawn perpendicular to a given plane?

(1) none (2) one (3) two (4) infinitely many

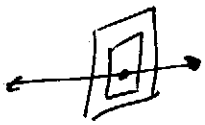
66) Which of the following statements is true?

(1) A line perpendicular to a plane is perpendicular to every line in the plane.

(2) The plane angles of a dihedral angle are never congruent.

(3) Two planes perpendicular to the same line at a given point coincide.

(4) Two planes perpendicular to the same plane intersect in exactly one line.



67) How many lines in space are perpendicular to a given line at a point on the line?

(1) none (2) one (3) two (4) infinitely many

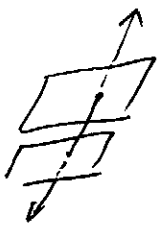
68) Which of the following statements is true?

(1) If a straight line is parallel to a plane, it is parallel to every line in the plane.

(2) If two planes are parallel, any line in one plane is parallel to any line in the other.

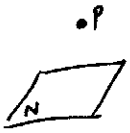
(3) Any two planes parallel to the same plane are perpendicular to each other.

(4) If a line intersects one of two parallel planes, then it intersects the other.



69) If point *P* lies outside of plane *N*, how many lines parallel to plane *N* can be drawn through point *P*?

(1) none (2) one (3) two (4) infinitely many



70) If point *P* lies outside of plane *N*, how many planes parallel to plane *N* can be drawn through point *P*?

(1) none (2) one (3) two (4) infinitely many