# Geometry R

## Unit 10 – Trigonometry

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**SIN, COS, & TAN RATIOS**

*Trigonometry* is the study of the properties of triangles. The word trigonometry means “angle measure”. A *trigonometric ratio* is a ratio of the lengths of two sides of a right triangle. The most common ratios are *sine* (sin), *cosine* (cos), and *tangent* (tan).

<table>
<thead>
<tr>
<th>Words</th>
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<tr>
<td>( \sin \angle A ) =</td>
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<td><img src="image" alt="Diagram" /></td>
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<td>( \cos \angle A ) =</td>
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<tr>
<td>( \tan \angle A ) =</td>
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**Examples**

1. Find the sine, cosine, and tangent of the following angles
   a. \( \angle P \)
   
   ![Diagram](image)

   b. \( \angle N \)

2. Find each value to the nearest ten thousandth.
   a. \( \sin 42^\circ \)

   b. \( \cos 65^\circ \)

   c. \( \tan 78^\circ \)
3. Find the missing measures. Round to the nearest tenth.

a. \[ \triangle ABC \]
   - \( A \)
   - \( B \)
   - \( C \)
   - \( 21 \text{ ft} \)
   - \( 36^\circ \)

b. \[ \triangle ABC \]
   - \( x \)
   - \( 32 \)
   - \( 53^\circ \)


c. \[ \triangle ABC \]
   - \( x \)
   - \( 73 \)
   - \( 21^\circ \)

4. The Leaning Tower of Pisa in Pisa, Italy, tilts about 5.2\(^\circ\) from vertical. If the tower is 55 meters tall, how far has its top shifted from its original position? Round to four decimal places.

   \[ \text{Height of the top of the tower} = 55 \text{ m} \times \sin(5.2^\circ) \]
5. From a point 120 m away from a building, Serena measures the angle between the ground and the top of a building and finds it measures 41°. What is the height of the building? Round to the nearest meter.

6. A cable anchors a utility pole to the ground as shown in the picture. The cable forms an angle of 70° with the ground. The distance from the base of the utility pole to the anchor point on the ground is 3.8 meters. Approximately how long, to the nearest meter, is the support cable?

7. Three city streets form a right triangle. Main Street and State Street are perpendicular. Laura Street and State Street intersect at a 50° angle. The distance along Laura Street to Main Street is 0.8 mile. If Laura Street is closed between Main Street and State Street for a festival, approximately how far (to the nearest tenth) will someone have to travel to get around the festival if they take only Main Street and State Street?
8. A shipmate set a boat to sail exactly $27^\circ$ NE from the dock. After traveling 120 miles, the shipmate realized he had misunderstood the instructions from the captain; he was supposed to set sail going directly east.

![Diagram of a ship sailing at $27^\circ$ NE from the dock, with a line segment representing 120 miles.

a. How many miles will they have to travel directly south before they are directly east of the Dock? Round your answer to the nearest mile.

b. How many extra miles did they travel by going the wrong direction compared to going directly east? Round your answer to the nearest mile.

9. A regular pentagon with side lengths of 14 cm is inscribed in a circle. What is the radius of the circle?
1. Dan was walking through a forest when he came upon a sizable tree. Dan estimated he was about 40 meters away from a tree when he measured the angle of elevation between the horizontal and the top of the tree to be 35 degrees. If Dan is about 2 meters tall, about how tall is the tree?

2. Dan was pretty impressed with this tree ... until he turned around and saw a bigger one, also 40 meters away but in the other direction. “Wow,” he said. “I bet that tree is at least 50 meters tall!” Then he thought a moment. “Hmm ... if it is 50 meters tall, I wonder what angle of elevation I would measure from my eye level to the top of the tree?” What angle will Dan find if the tree is 50 meters tall? Explain your reasoning.

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Given a right triangle $\triangle ABC$, the measure of angle $C$ can be found in the following ways:

- $\arcsin \left( \frac{AB}{AC} \right) = \angle C$
- $\arccos \left( \frac{BC}{AC} \right) = \angle C$
- $\arctan \left( \frac{AB}{BC} \right) = \angle C$

We can write similar statements to determine the measure of angle $A$.

We can use a calculator to help us determine the values of arcsin, arccos, and arctan. Most calculators show these buttons as “$\sin^{-1}$,” “$\cos^{-1}$,” and “$\tan^{-1}$.”
3. Find the measure of angles a–d to the nearest degree.

   a.  
   
   b.  
   
   c.  
   
   d.  

4. Shelves are being built in a classroom to hold textbooks and other supplies. The shelves will extend 10 in from the wall. Support braces will need to be installed to secure the shelves. The braces will be attached to the end of the shelf and secured 6 in below the shelf on the wall. What angle measure will the brace and the shelf make?
5. A 16 ft ladder leans against a wall. The foot of the ladder is 7 ft from the wall.
   a. Find the vertical distance from the ground to the point where the top of the ladder touches the wall.
   b. Determine the measure of the angle formed by the ladder and the ground.

6. A group of friends have hiked to the top of the Mile High Mountain. When they look down, they can see their campsite, which they know is approximately 3 miles from the base of the mountain.
   a. Sketch a drawing of the situation.
   b. What is the angle of depression?
7. A roller coaster travels 80 ft of track from the loading zone before reaching its peak. The horizontal distance between the loading zone and the base of the peak is 50 ft.
   a. Model the situation using a right triangle.

   b. At what angle is the roller coaster rising according to the model?
APPLYING TANGENTS

Angle of elevation
- the angle from the horizontal measured upward to the line of sight of the observer.

Angle of depression
- the angle from the horizontal measured downward to the line of sight of the observer.

1. Scott, whose eye level is 1.5 m above the ground, stands 30 m from a tree. The angle of elevation of a bird at the top of the tree is 36°. How far above ground is the bird? Round to the nearest hundredth.
2. From an angle of depression of 40°, John watches his friend approach his building while standing on the rooftop. The rooftop is 16 m from the ground, and John's eye level is at about 1.8 m from the rooftop. What is the distance between John's friend and the building, to the nearest tenth?

3. Standing on the gallery of a lighthouse (the deck at the top of a lighthouse), a person spots a ship at an angle of depression of 20°. The lighthouse is 28 m tall and sits on a cliff 45 m tall as measured from sea level. What is the horizontal distance between the lighthouse and the ship? Round to the nearest meter.

4. Samuel is at the top of a tower and will ride a trolley down a zip-line to a lower tower. The total vertical drop of the zip-line is 40 ft. The zip line's angle of elevation from the lower tower is 11.5°. What is the horizontal distance between the towers, to the nearest foot?
5. Find to the nearest foot the height of a tree that casts a 24-foot shadow when the angle of elevation of the sun is $52^\circ$. 
COFUNCTIONS

In right triangle $ABC$, the measurement of acute angle $\angle A$ is denoted by $\alpha$, and the measurement of acute angle $\angle B$ is denoted by $\beta$. Determine the following values in the table:

<p>| | | | |</p>
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<thead>
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<tbody>
<tr>
<td>$\sin \alpha$</td>
<td>$\sin \beta$</td>
<td>$\cos \alpha$</td>
<td>$\cos \beta$</td>
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What can you conclude from the results?

1. Consider the right triangle $ABC$ so that $\angle C$ is a right angle, and the degree measures of $\angle A$ and $\angle B$ are $\alpha$ and $\beta$, respectively.
   
   a. Find $\alpha + \beta$.
   
   b. Use trigonometric ratios to describe $\frac{BC}{AB}$ two different ways.
   
   c. Use trigonometric ratios to describe $\frac{AC}{AB}$ two different ways.
   
   d. What can you conclude about $\sin \alpha$ and $\cos \beta$?
   
   e. What can you conclude about $\cos \alpha$ and $\sin \beta$?
2. Find values for $\theta$ that make each statement true.
   
   a. $\sin \theta = \cos (25^\circ)$
   
   b. $\sin 80^\circ = \cos \theta$
   
   c. $\sin \theta = \cos (\theta + 10^\circ)$
   
   d. $\sin (\theta - 45^\circ) = \cos (\theta)$

3. For what angle measurement must sine and cosine have the same value? Explain how you know.

4. What is happening to $a$ and $b$ as $\theta$ changes? What happens to $\sin \theta$ and $\cos \theta$?
There are certain special angles where it is possible to give the exact value of sine and cosine. These are the angles that measure 0°, 30°, 45°, 60°, and 90°; these angle measures are frequently seen.

You should memorize the sine and cosine of these angles with quick recall just as you did your arithmetic facts.

a. Learn the following sine and cosine values of the key angle measurements.

<table>
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<tr>
<th>θ</th>
<th>0°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>90°</th>
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<tr>
<td>Sin</td>
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b. △ABC is equilateral, with side length 2; D is the midpoint of side AC. Label all side lengths and angle measurements for △ABD. Use your figure to determine the sine and cosine of 30° and 60°.

c. Draw an isosceles right triangle with legs of length 1. What are the measures of the acute angles of the triangle? What is the length of the hypotenuse? Use your triangle to determine sine and cosine of the acute angles.
5. Find the missing side lengths in the triangle.

\[ \triangle \text{with sides } 3 \text{ and } a \text{ and angle } 30^\circ \]

6. Find the missing side lengths in the triangle.

\[ \triangle \text{with sides } 3 \text{ and } c \text{ and angle } 30^\circ \]
AREA OF A TRIANGLE

Three triangles are presented below. Determine the areas for each triangle, if possible. If it is not possible to find the area with the provided information, describe what is needed in order to determine the area.

What if the third side length of the triangle were provided? Is it possible to determine the area of the triangle now? Find the area of \( \triangle GHI \).
Now consider \( \triangle ABC \) which is set up similarly to the triangle from the last example.

What is the area of this triangle?
Area of a Triangle

The formula $K = \frac{1}{2}bh$ is for the area of a triangle when the base and height are given.

The following formula can be used when the height is not given.
The area of a triangle is equal to one-half the product of the measures of two sides and the sine of the angle between them.

To use this formula you NEED TWO SIDES AND THE INCLUDED ANGLE

\[
\text{Area of } \triangle ABC = \frac{1}{2}ab \sin C \\
= \frac{1}{2}ac \sin B \\
= \frac{1}{2}bc \sin A
\]

1. Find the area of $\triangle BAD$ if $BA = 8$, $AD = 6$, and $m \angle A = 150^\circ$.

2. Find to the nearest hundred the number of square feet in the area of a triangular lot at the intersection of two streets of the angle of intersection is $76^\circ 10'$ and the frontage along the streets are 220 feet and 156 feet.

3. The area of a parallelogram is 20. Find the measure of the angles of the parallelogram if the measures of two adjacent sides are 8 and 5.
4. Find the *exact value* of the area of an equilateral triangle if the measure of one side is 4

5. A farmer is planning how to divide his land for planti}ng next year’s crops. A triangular plot of land is left with two known side lengths measuring 500 m and 1,700 m.

What could the farmer do next in order to find the area of the plot?

6. A real estate developer is searching for his next piece of land to build on. He examines a plot of land in the shape of △ *ABC*. The real estate developer measures the length of *AB* and *AC* and finds them to both be approximately 4,000 feet, and the included angle has a measure of approximately 50°.

a. Draw a diagram that models the situation, labeling all lengths and angle measures. Find the area of the piece of land.
a. Find the lengths of $d$ and $e$.

b. Find the lengths of $x$ and $y$. How is this different from part (a)?

We will show how two facts in trigonometry aid us to find unknown measurements in triangles that are not right triangles.

a. What is $\sin \angle C$?

b. What is $\sin \angle A$?

c. What is $\sin \angle B$?
The Law of Sines is a proportion that relates the length of any two sides of a triangle to the sine of the angles opposite these sides.

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

1. A surveyor needs to determine the distance between two points \(A\) and \(B\) that lie on opposite banks of a river. A point \(C\) is chosen 160 meters from point \(A\), on the same side of the river as \(A\). The measures of angles \(\angle BAC\) and \(\angle ACB\) are 41° and 55°, respectively. Approximate the distance from \(A\) to \(B\) to the nearest meter.

2. In \(\triangle ABC\), \(m\angle A = 30\), \(a = 12\), and \(b = 10\). Find the measure of \(\angle B\). Include a diagram in your answer.

3. A car is moving towards a tunnel carved out of the base of a hill. As the accompanying diagram shows, the top of the hill, \(H\), is sighted from two locations, \(A\) and \(B\). The distance between \(A\) and \(B\) is 250 ft. What is the height, \(h\), of the hill to the nearest foot?
4. Given \( \triangle ABC \), \( AB = 14 \), \( \angle A = 57.2^\circ \), and \( \angle C = 78.4^\circ \), calculate the measure of angle \( B \) to the nearest tenth of a degree, and use the Law of Sines to find the lengths of \( AC \) and \( BC \) to the nearest tenth. Calculate the area of \( \triangle ABC \) to the nearest square unit.

5. Given \( \triangle DEF \), \( \angle F = 39^\circ \), and \( EF = 13 \), calculate the measure of \( \angle E \), and use the Law of Sines to find the lengths of \( DF \) and \( DE \) to the nearest hundredth.

6. Does the law of sines apply to a right triangle?
7. Given quadrilateral $GHJK$, $\angle H = 50^\circ$, $\angle HKG = 80^\circ$, $\angle KGJ = 50^\circ$, $\angle J$ is a right angle and $GH = 9\text{ in.}$, use the law of sines to find the length of $GK$, and then find the lengths of $GJ$ and $JK$ to the nearest tenth of an inch.

8. If $r = 8$, $q = 7$, and $m\angle R = 52^\circ$, determine the measures of each angle in triangle $PQR$. Round to the nearest degree.

9. Two forces act on a body, making angles of $16^\circ$ and $37^\circ$ with the resultant. If the larger force is 42 pounds, what is the magnitude of the resultant to the nearest pound.
DOUBLE TRIANGLE PROBLEMS

Use the Laws of Sines to find all missing side lengths for each of the triangles in the Exercises below. Round your answers to the tenths place.

1. Use the triangle below to complete this exercise.
   
   a. Find the lengths of $\overline{AC}$ and $\overline{AB}$.

![Triangle Diagram]

2. Two lighthouses are 30 mi. apart on each side of shorelines running north and south, as shown. Each lighthouse keeper spots a boat in the distance. One lighthouse keeper notes the location of the boat as 40° east of south, and the other lighthouse keeper marks the boat as 32° west of south. What is the distance from the boat to each of the lighthouses at the time it was spotted? Round your answers to the nearest mile.

![Lighthouse Diagram]
3. A ship captain at sea uses a sextant to sight an angle of elevation of 37° to the top of a lighthouse. After the ship travels 250 feet directly towards the lighthouse, another sighting is made, and the new angle of elevation is 50°. The ship’s charts show that there are dangerous rocks 100 feet from the base of the lighthouse. Find, to the nearest foot, how close to the rocks the ship is at the time of the second sighting.

4. A young tree is braced by two wires extending in straight lines from the same point on the trunk of the tree to points on the ground on opposite sides of the tree. The wires are fastened to stakes on the ground 64 inches apart and make angles of 35° and 40° with the ground. Find, to the nearest inch, how far up the tree the wires are attached.