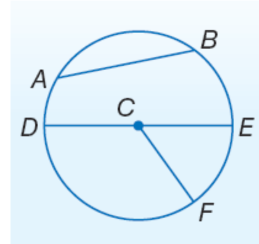
GEOMETRY R

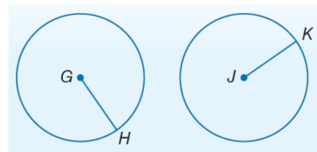
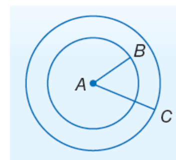
Unit 13 – Circles

Date	Classwork	Day	Assignment
Thursday 4/4	**Unit 12 Test**		
Friday 4/5	Central angles Inscribed Angles	1	HW 13.1
Monday 4/8	Central angles Inscribed Angles	1	HW 13.1
Tuesday 4/9	Arcs and Chords	2	HW 13.2
Wednesday 4/10	Area of Sectors and Area problems Arc Length	3	HW 13.3
Thursday 4/11	Tangents ***Unit 13 Quiz 1***	4	HW 13.4
Friday 4/12	Secants, Tangents, and Angle Measures	5	HW 13.5
Monday 4/15	Lengths of Secant Segments, Tangent Segments, and Chords	6	HW 13.6
Tuesday 4/16	Writing Equations of Circles (Standard Form) ***Unit 13 Quiz 2***	7	HW 13.7
Wednesday 4/17	Equations of Circles (General Form)	8	HW 13.8
Thursday 4/18	Inscribed and Circumscribed Polygons ***Unit 13 Quiz 3***	9	HW 13.9
4/19 – 4/26	*No School*		
Monday 4/29	Review	10	Review Sheet
Tuesday 4/30	Review	11	Review Sheet
Wednesday 5/1	Review	12	Review Sheet
Thursday 5/2	**Unit 13 Test**	13	

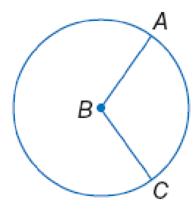
***CIRCLES, ARCS, ANGLES, AND CHORDS* [1]**

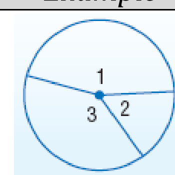
*A **circle** is the locus or set of all points in a plane equidistant from a given point called the **center** of the circle.*



Words	*Figure*
Two circles are **congruent circles** if and only if they have congruent radii.	
Concentric Circles are coplanar circles that have the same center.	

*A **central angle** of a circle is an angle whose vertex is the center of the circle. Its sides contain two radii of the circle.*

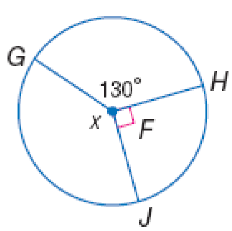


Sum of Central Angles	
Words	*Example*
The sum of the measures of the central angles of a circle with no interior points in common is 360 degrees.	

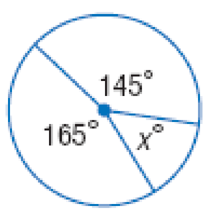
Examples

Find the value of x.

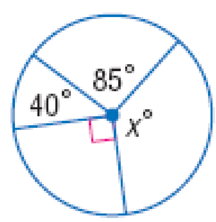
a.



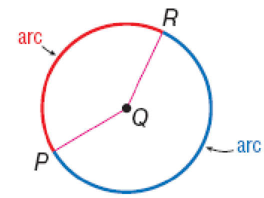
b.

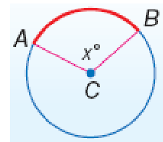
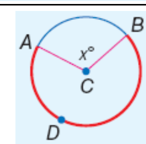
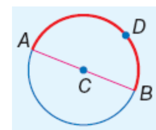
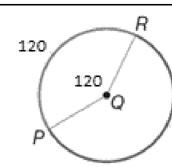


c.



An ***arc*** *is a portion of a circle defined by two endpoints. A central angle separates the circle into two arcs with measures related to the measure of the central angle.*

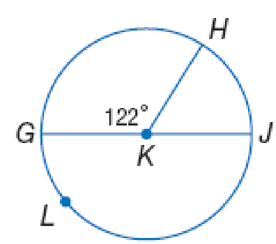


Arc	*Figure*
Minor Arc is the shortest arc connecting two endpoints on a circle	
Major Arc is the longest arc connecting two endpoints on a circle	
Semicircle is an arc with endpoints that lie on a diameter	
The measure of an arc of a circle is equal to the measure of its corresponding central angle.	

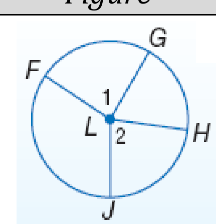
Examples

$\overline{GJ}$ *is a diameter of circle K. Identify each arc as a major arc, minor arc, or semicircle. Then find its measure.*

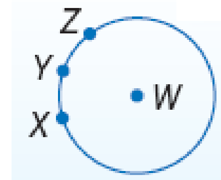
a. $m\,GH$ b. $m\,GLH$ c. $m\,GLJ$



Congruent arcs *are arcs in the same or congruent circles that have the same measure.*

Words	*Example*	*Figure*
In the same circle or in congruent circles, two minor arcs are congruent if and only if their central angles are congruent.		

__Adjacent arcs__ are arcs that have exactly one point in common.

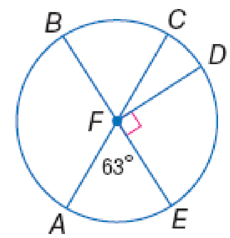
Arc Addition Postulate		
Words	*Example*	*Figure*
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.		

Examples

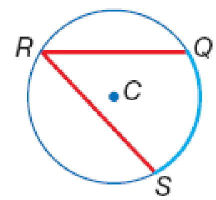
Find each measure in circle F.

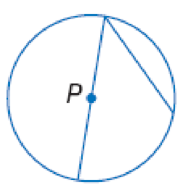
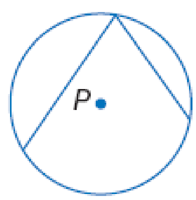
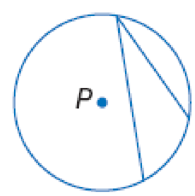
a. $m\widehat{AED}$

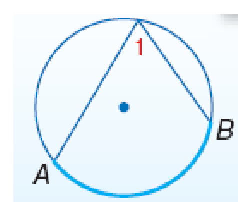
b. $m\widehat{ADB}$



An __inscribed angle__ has a vertex on a circle and sides that contain chords of the circle. An __intercepted arc__ has endpoints on the sides of an inscribed angle and lies in the interior of the inscribed angle.



Case 1	*Case 2*	*Case 3*
		

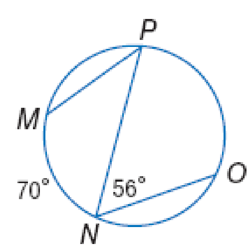
Inscribed Angle Theorem		
Words	*Example*	*Figure*
If an angle is inscribed in a circle, then the measure of the angle equals one half the measure of its intercepted arc.		

Examples

Find each measure.

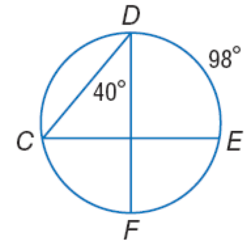
a. $m\angle P$

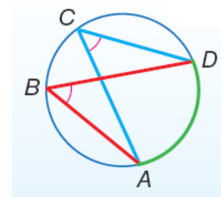
b. $m\widehat{PO}$



c. $m\widehat{CF}$

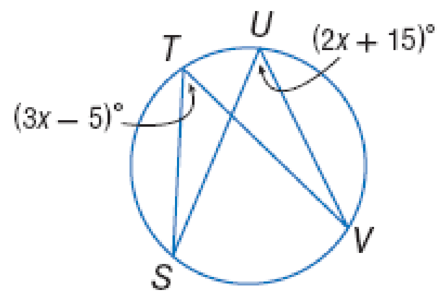
d. $m\angle C$



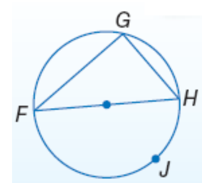
Words	*Example*	*Figure*
If two inscribed angles of a circle intercept the same arc or congruent arcs, then the angles are congruent.		

Examples

a. Find $m\angle T$

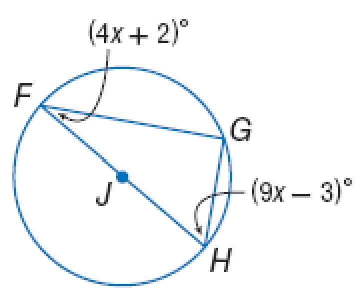


b. If $m\angle S = 3x$ and $m\angle V = x + 16$, find $m\angle S$.

Thales' Theorem		
Words	*Examples*	*Figures*
An inscribed angle of a triangle intercepts a diameter or semicircle if and only if the angle is a right angle.		

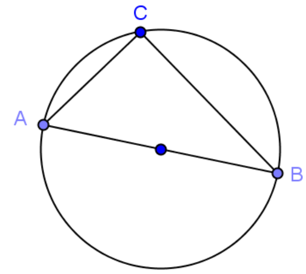
Examples

1. a. Find $m\angle F$.



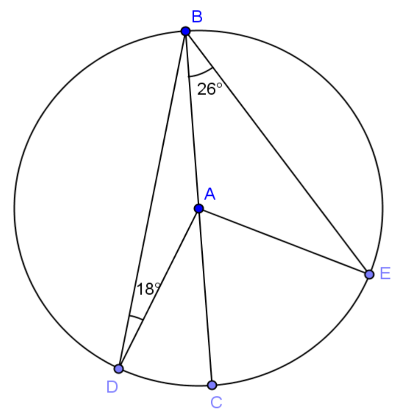
b. If $m\angle F = 7x + 2$ and $m\angle H = 17x - 8$, find x.

2. In the figure below, $\overline{\boldsymbol{AB}}$ is the diameter of a circle of radius $\mathbf{17}$ miles. If $\boldsymbol{BC} = \mathbf{30}$ miles, what is $\boldsymbol{AC}$?



3. In the circle shown, $\overline{BC}$ is a diameter with center A.

   a. Find $m\angle DAB$.

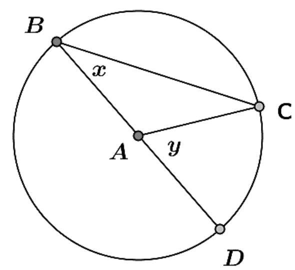


   b. Find $m\angle BAE$.

   c. Find $m\angle DAE$.

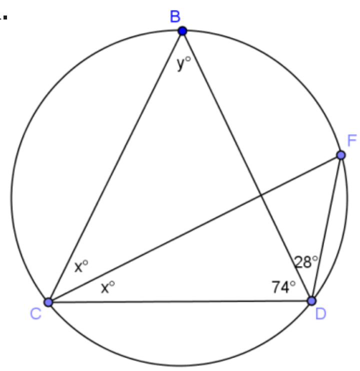
4. Rectangle $ABCD$ is inscribed in circle P. Boris says that diagonal AC could pass through the center, but it does not have to pass through the center. Is Boris correct? Explain your answer in words, or draw a picture to help you explain your thinking.

5. Prove that $y = 2x$ in the diagram provided.

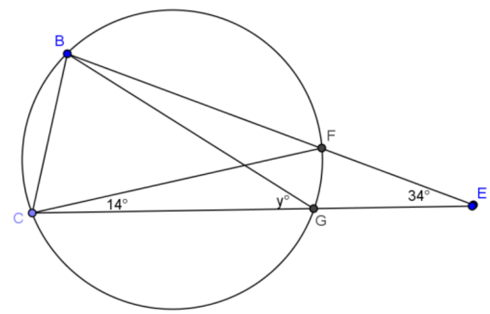


6. Find the measures of the labeled angles.

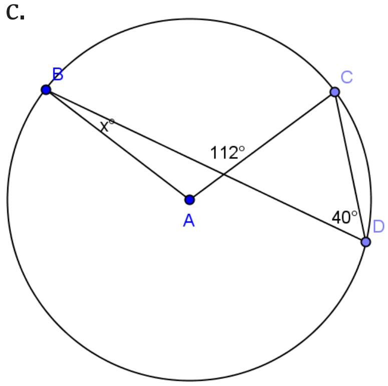
a.



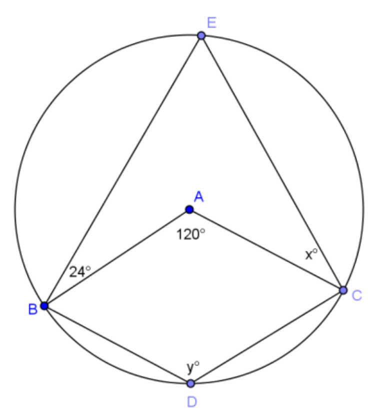
b.



c.

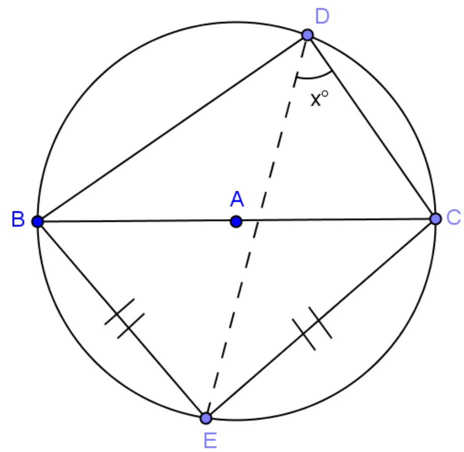


d.

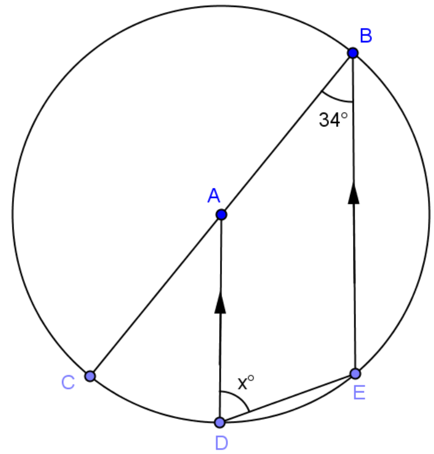


Find the value x in each figure below, and describe how you arrived at the answer.

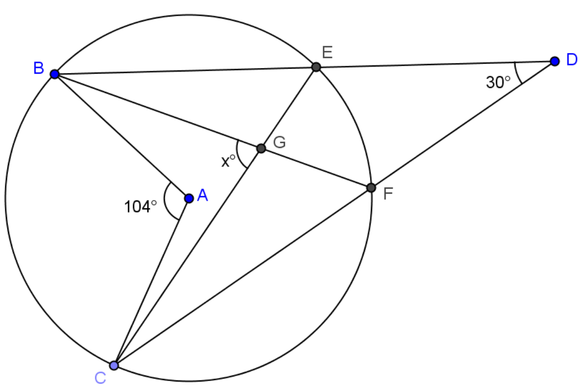
1. Hint: Thales' theorem



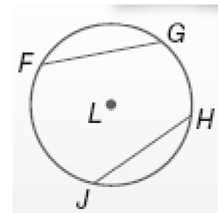
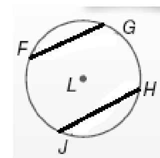
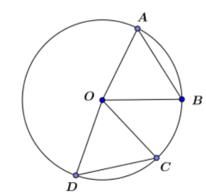
2.



3.

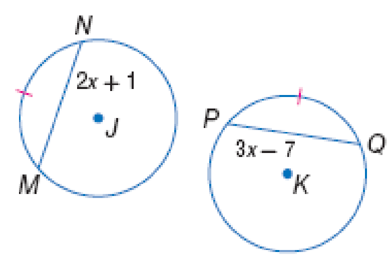


ARCS AND CHORDS [2]

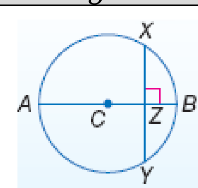
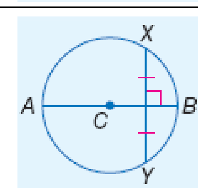
Words	*Example*
In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent	
Arcs between parallel chords are congruent.	
In a circle, central angles are congruent if and only if their corresponding chords are congruent.	

Examples

a. In the figures, circle J ≅ circle K and $MN \cong PQ$. Find *PQ*.

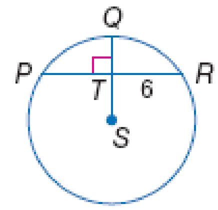


If a line, segment, or ray divides an arc into two congruent arcs, then it ***bisects*** *the arc.*

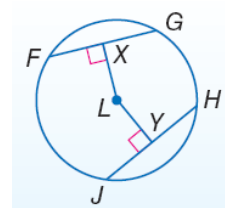
Bisecting Arcs and Chords		
Words	*Examples*	*Figures*
If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc.		
If a diameter (or radius) of a circle bisects a chord, then it must be perpendicular to the chord.		

Examples

a. In circle S, $m\widehat{PQR} = 98$. Find $m\widehat{PQ}$.

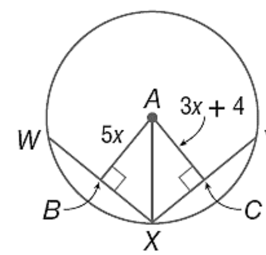


b. In circle S, find *PR*.

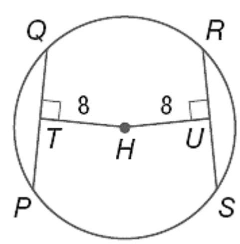
Words	*Example*	*Figure*
In the same circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.		

Examples

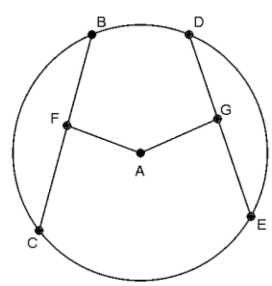
1. In circle A, *WX* = *XY* = 22. Find *AB*.



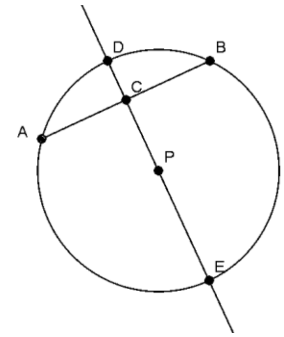
2. In circle H, *PQ* = 3*x* – 4 and *RS* = 14. Find *x*.



3. Given circle A shown, $AF = AG$ and $BC = 22$. Find DE.

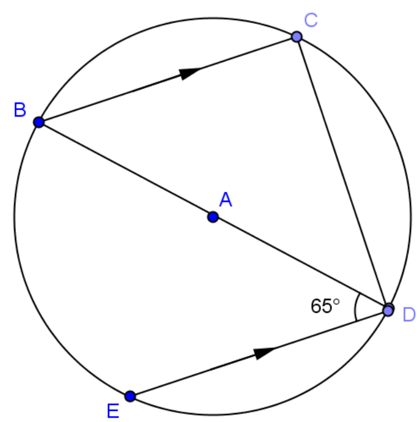


4. In the figure, circle P has a radius of 10. $\overline{AB} \perp \overline{DE}$.
   a. If $AB = 8$, what is the length of AC?



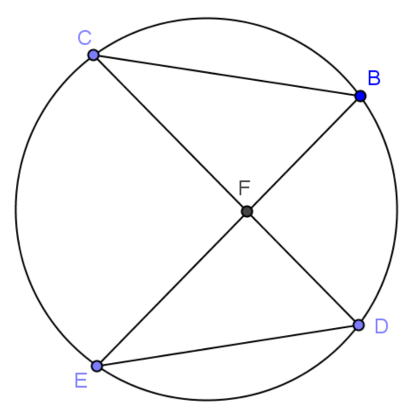
   b. If $DC = 2$, what is the length of AB?

5. Find the angle measure of $\widehat{CD}$ and $\widehat{ED}$.



6. $m\widehat{CB} = m\widehat{ED}$ and $m\widehat{BC}: m\widehat{BD}: m\widehat{EC} = 1: 2: 4$. Find

a. $m\angle BCF$

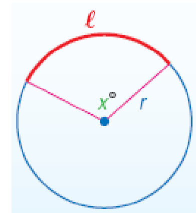


b. $m\angle EDF$

c. $m\angle CFE$

ARC LENGTH AND AREA OF SECTORS [3]

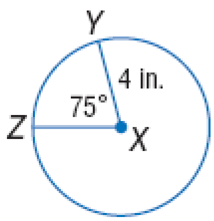
Arc length *is the distance between the endpoints along an arc measured in linear units. Since an arc is a portion of a circle, its length is a fraction of the circumference.*

Arc Length		
Words	*Proportion*	*Figure*
The ratio of the length of an arc L to the circumference of the circle is equal to the ratio of the degree measure of the arc to 360.		

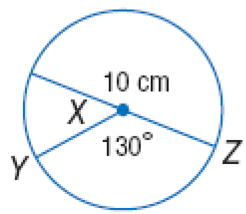
Examples

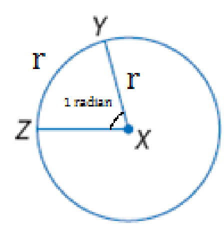
Find the length of $\widehat{ZY}$. Round to the nearest hundredth.

a.



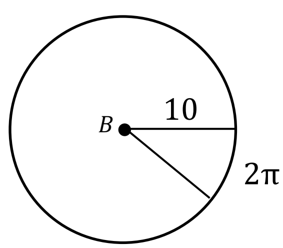
b.



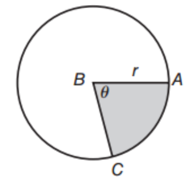
Radian Measure		
A radian is the measure of the central angle of a sector of a circle with arc length of one radius length.	π radians = 180° To convert degrees to radians, multiply by $\frac{\pi}{180°}$	

1. Convert 120° to radians.

2. What is the measure of angle *B*, in radians?

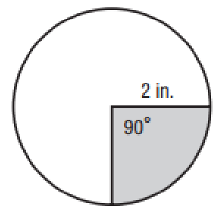


The area of a circle is found by using the formula $A = \pi r^2$. A ***sector*** *is a pie-shaped portion of the circle enclosed by 2 radii and the edge of the circle. A central angle of a sector is an angle whose vertex is at the center of the circle and crosses the circle.*

Area of Sectors		
Words	*Proportion*	*Figure*
The area of a sector θ is proportional to the part that the central angle is of 360°.	$\frac{area\ of\ sector}{area\ of\ circle} = \frac{\theta}{360}$	

Examples

a. Find the area of the sector shown at the right.



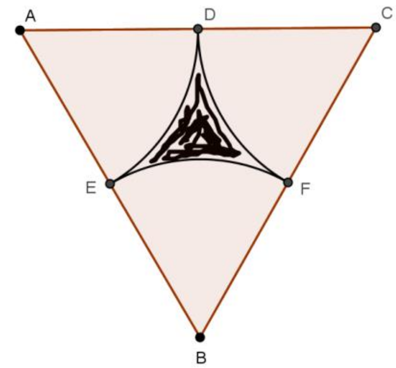
b. Find the area of a sector if the circle has a radius of 10 centimeters and the central angle measures 72.

c. Find the area of a sector if the circle has a radius of 5 inches and the central angle measures 60.

d. If the area of a sector is 15π square centimeters and the radius of the circle is 5 centimeters, find the measure of the central angle.

e. Find the measure of the central angle that intercepts a sector that is $\frac{1}{3}$ the area of the circle.

1. ΔABC is an equilateral triangle with edge length 20 cm. D, E, and F are midpoints of the sides. The vertices of the triangle are the centers of the circles creating the arcs shown. Find the following (round to the nearest hundredth):



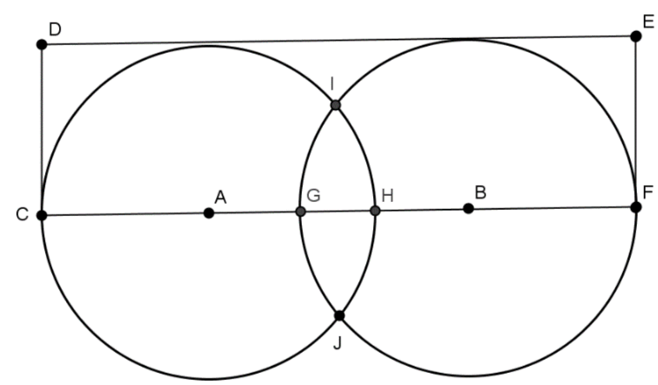
   a. The area of the sector with center A.

   b. The area of triangle ABC.

   c. The area of the shaded region.

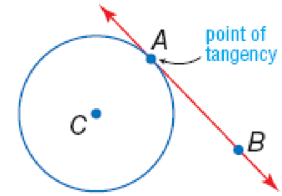
   d. The perimeter of the shaded region.

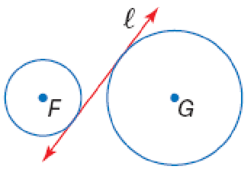
2. In the figure shown, $AC = BF = 5$ cm, $GH = 2$ cm, and $m\angle\text{HBI} = 30°$. Find the area in the rectangle, but outside of the circles (round to the nearest hundredth).

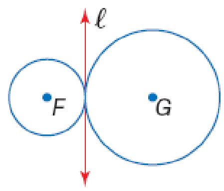


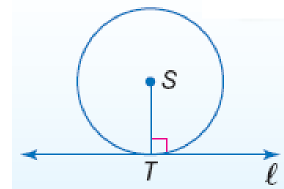
TANGENTS [4]

A ***tangent*** *is a line in the same plane as a circle that intersects the circle in exactly one point, called the* ***point of tangency****. A* ***common tangent*** *is a line, ray, or segment that is tangent to two circles in the same plane.*



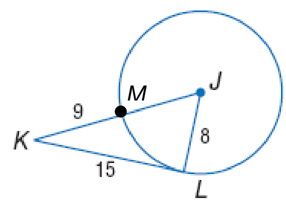




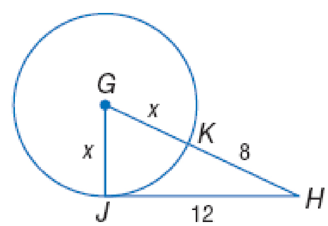
Theorem		
Words	*Example*	*Figure*
In a plane, a line is tangent to a circle if and only if it is perpendicular to a radius drawn to the point of tangency.		

Examples

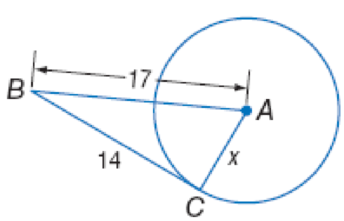
a. $\overline{JL}$ is a radius of circle *J*. Determine whether $\overline{KL}$ is tangent to circle *J*. Justify your answer.

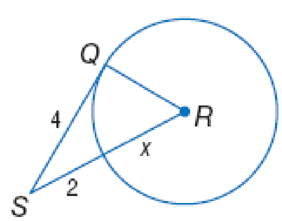


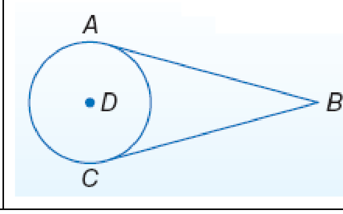
b. $\overline{JH}$ is tangent to circle *G* at *J*. Find the value of *x*.



c. Find the value of *x*. Assume that segments that appear to be tangent are tangent.

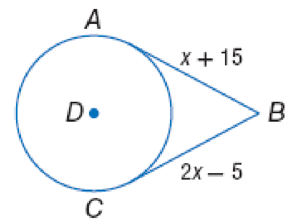




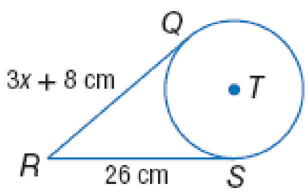
Theorem		
Words	*Example*	*Figure*
If two segments from the same exterior point are tangent to a circle, then they are congruent.		

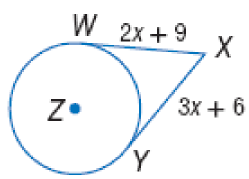
Examples

a. $\overline{AB}$ and $\overline{CB}$ are tangent to circle *D*. Find the value of *x*.

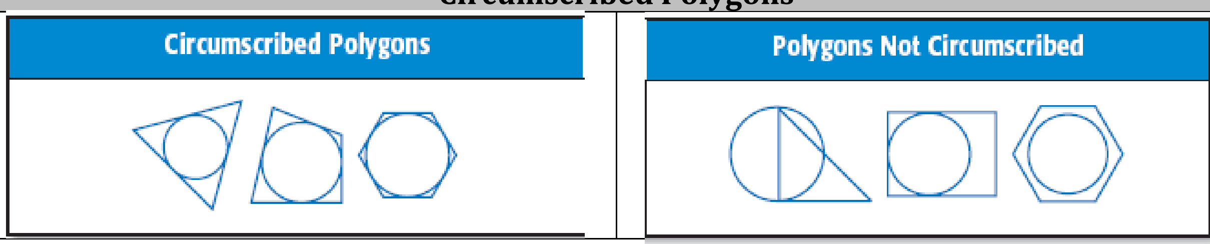


b. Find the value of *x*. Assume that segments that appear to be tangent are tangent.



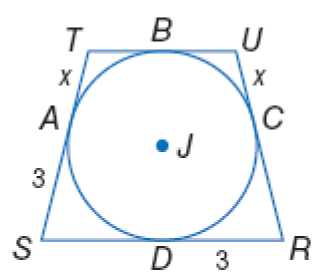


Circumscribed Polygons



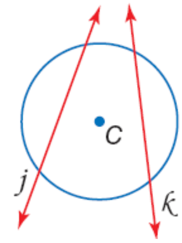
Examples

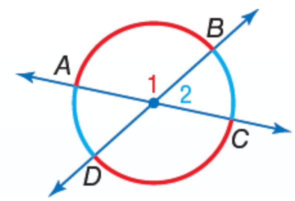
a. Quadrilateral *RSTU* is circumscribed about circle *J*. If the perimeter is 18 units, find *x*.



SECANTS, TANGENTS, AND ANGLE MEASURES [5]

*A **secant** is a line that intersects a circle in exactly two points. When two secants intersect inside a circle, the angles formed are related to the arcs they intercept.*

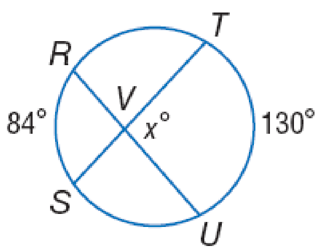


Theorem		
Words	*Example*	*Figure*
If two secants or chords intersect in the interior of a circle, then the measure of an angle formed is one half the sum of the measure of the arcs intercepted by the angle and its vertical angle.		

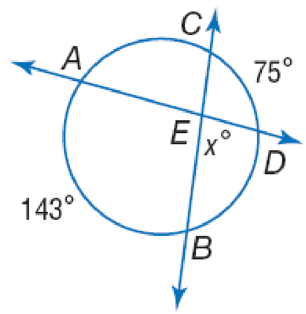
Examples

Find x.

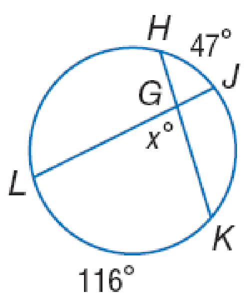
a.



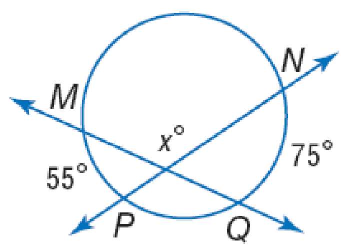
b.

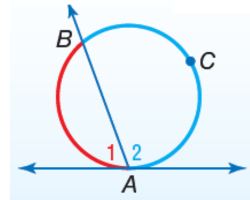


c.



d.

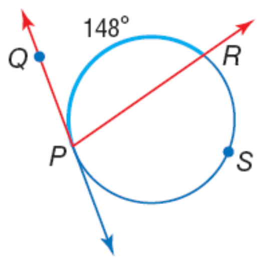


Theorem		
Words	*Example*	*Figure*
If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one half the measure of its intercepted arc.		

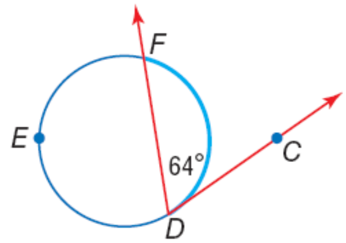
Examples

Find each measure.

a. $m\angle QPR$

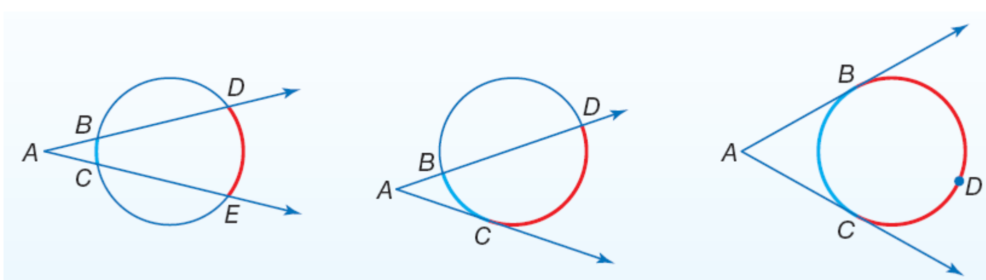


b. $m\angle DEF$



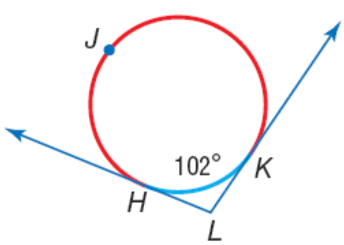
Theorem
If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is half the difference of the measures of the intercepted arcs.

Examples

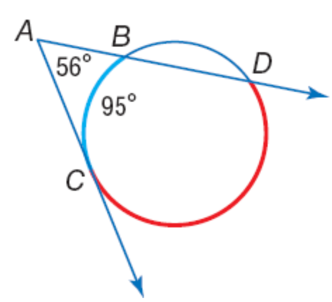


Find each measure.

a. $m\angle L$

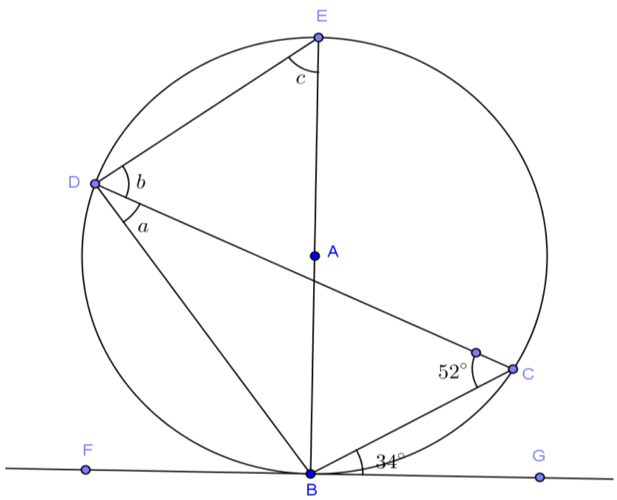


b. $m\widehat{CD}$

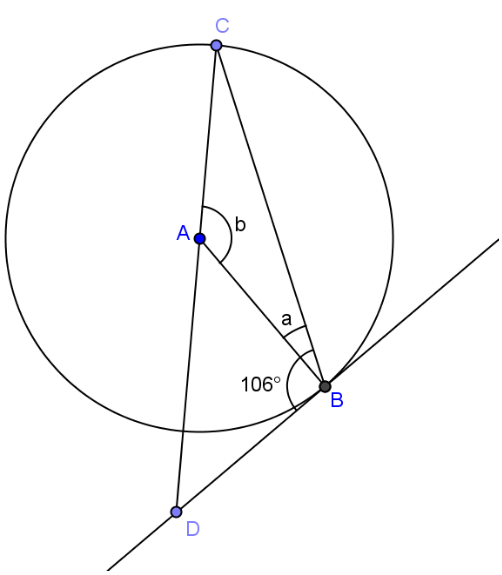


Find $x, y, a, b,$ and/or c.

1.

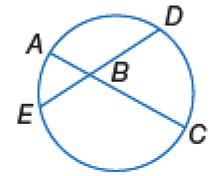


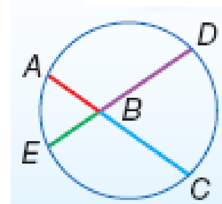
2.



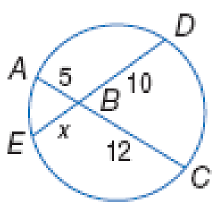
LENGTHS OF TANGENT SEGMENTS, SECANT SEGMENTS, AND CHORDS [6]

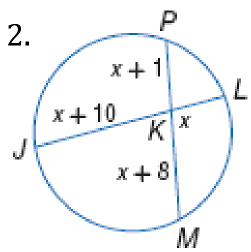
When two chords intersect inside a circle, each chord is divided into two segments, called ***chord segments****. In the figure, chord* $\overline{AC}$ *is divided into segments* $\overline{AB}$ *and* $\overline{BC}$*. Likewise, chord* $\overline{ED}$ *is divided into segments* $\overline{EB}$ *and* $\overline{BD}$*.*

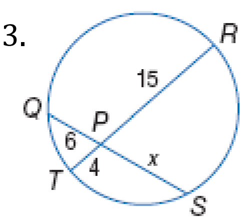


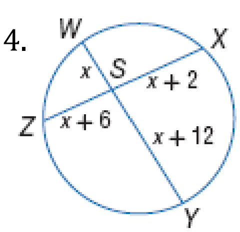
Theorem		
Words	*Example*	*Figure*
If two chords intersect in a circle, then the products of the lengths of the chord segments are equal.		

Examples
Find x.

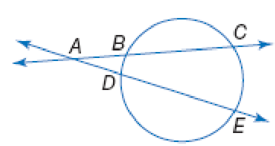
1.


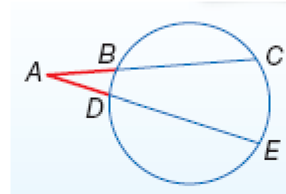
2.


3.


4.


A ***secant segment*** *is a segment of a secant line that has exactly one endpoint on the circle. A secant segment that lies in the exterior of the circle is called an* ***external secant segment****.*

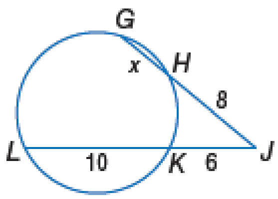


Theorem		
Words	*Example*	*Figure*
If two secants intersect in the exterior of a circle, then the product of the measures of one secant segment and its external secant segment is equal to the product of the measures of the other secant and its external secant segment.		

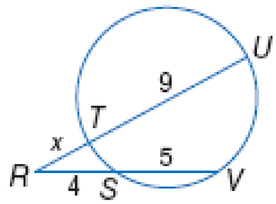
Examples

Find x.

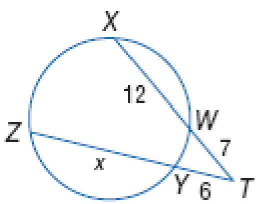
1.



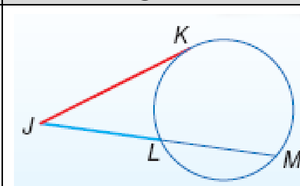
2.



3.

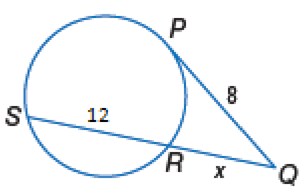


*An equation similar to the one above can be used when a secant and a tangent intersect outside a circle. In this case, the **tangent segment**, or segment of a tangent with one endpoint on the circle, is both the exterior and whole segment.*

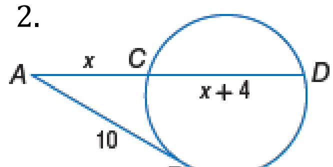
Theorem		
Words	*Example*	*Figure*
If a tangent and a secant intersect in the exterior of a circle, then the square of the measure of the tangent is equal to the product of the measures of the secant and its external secant segment.		

Find x.

1.

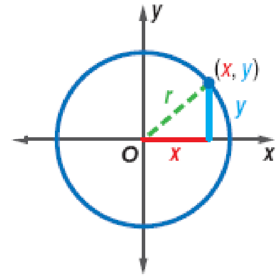


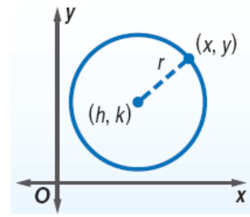
2.



EQUATIONS OF CIRCLES (STANDARD FORM) [7]

Since all points on a circle are equidistant from the center, you can find an equation of a circle by using the Distance Formula.



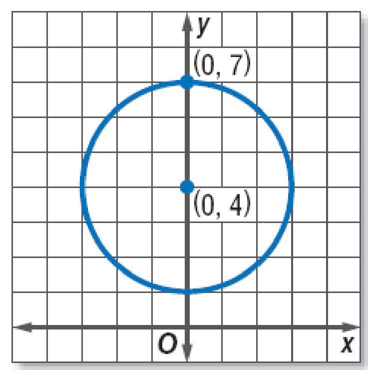
Equation of a Circle – Center-Radius/Standard Form	
Words	*Figure*
The standard form of the equation of a circle with center (h, k) and radius r, is $(x - h)^2 + (y - k)^2 = r^2$. (Center-Radius Form)	

Examples

Write the equation of each circle.

a. center at (1, -8), radius 7

b. the circle graphed at the right



c. center at origin, radius $\sqrt{10}$

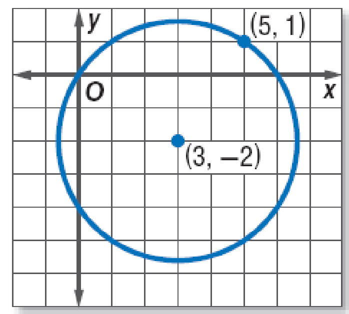
d. Center at (4, -1), diameter 8

Examples

Write the equation of each circle.

a. center at (-2, 4), passes through (-6, 7)

b. the circle graphed at the right



c. center at (5, 4), passes through (-3, 4)

d. center at (-3, -5), passes through (0, 0)

e. The equation of a circle is $(x - 4)^2 + (y + 1)^2 = 9$. State the center and the radius. Then graph the equation.

EQUATIONS OF CIRCLES (GENERAL FORM) [8]

a. Express $(x - 5)^2$ as a trinomial.

b. Express $(x + 4)^2$ as a trinomial.

c. Factor the trinomial: $x^2 + 12x + 36$.

d. Complete the square to solve the following equation: $x^2 + 6x - 40 = 0$
(Use a visual representation to support your work).

Sometimes equations of circles are presented in simplified form. To easily identify the center and the radius of the circle, we sometimes need to factor and/or complete the square in order to rewrite the equation in its center-radius or standard form.

1. Rewrite the following equations in the form $(x - h)^2 + (y - k)^2 = r^2$.
   a. $x^2 + 4x + 4 + y^2 - 6y + 9 = 36$

   b. $x^2 - 10x + 25 + y^2 + 14y + 49 = 4$

2. What is the center and radius of the following circle?
   $x^2 + 4x + y^2 - 12y - 41 = 0$

3. Identify the center and radius for each of the following circle.
   a. $x^2 - 20x + y^2 + 6y = 35$

   b. $x^2 - 3x + y^2 - 5y = \frac{19}{2}$

4. Could the circle with equation $x^2 - 6x + y^2 - 7 = 0$ have a radius of 4? Why or why not?

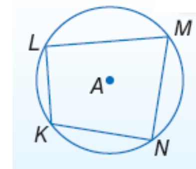
5. Stella says the equation $x^2 - 8x + y^2 + 2y = 5$ has a center of (4, -1) and a radius of 5. Is she correct? Why or why not?

6. Identify the graphs of the following equations as a circle, a point, or an empty set.
   a. $x^2 + y^2 + 4x = 0$

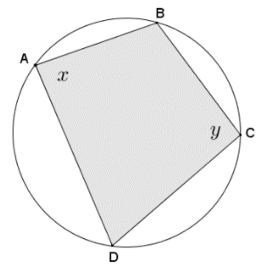
   b. $x^2 + y^2 + 6x - 4y + 15 = 0$

   c. $2x^2 + 2y^2 - 5x + y + \frac{13}{4} = 0$

INSCRIBED QUADRILATERALS [9]

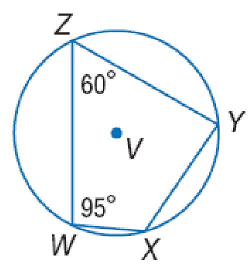
Theorem		
Words	*Example*	*Figure*
If a quadrilateral is inscribed in a circle (cyclic quadrilateral), then its opposite angles are supplementary.		

Given: Cyclic quadrilateral *ABCD*
Prove: $x + y = 180°$

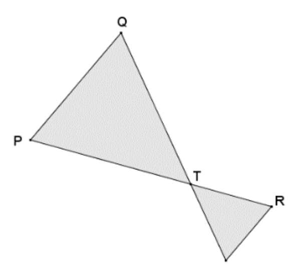


Examples

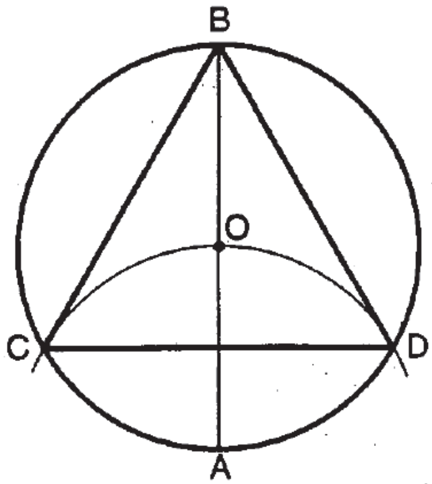
a. Quadrilateral *WXYZ* is inscribed in circle *V*. Find $m\angle X$ and $m\angle Y$.



b. Quadrilateral *PQRS* is a cyclic quadrilateral. Explain why $\Delta PQT \sim \Delta SRT$.

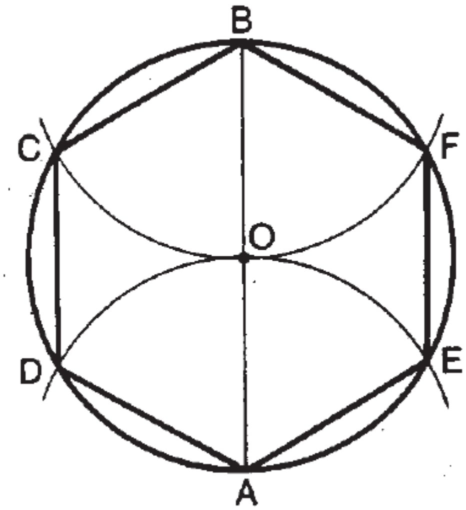


CONSTRUCTING INSCRIBED POLYGONS



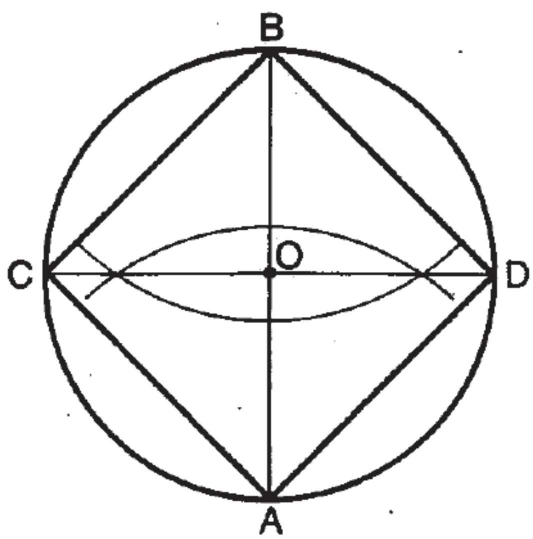
INSCRIBE AN EQUILATERAL TRIANGLE

1. In the given circle O, draw a diameter $\overline{AB}$.
2. Using A as a center and AO as a radius, draw an arc intersecting the circle at C and D.
3. Connect B, C, and D to form the triangle.



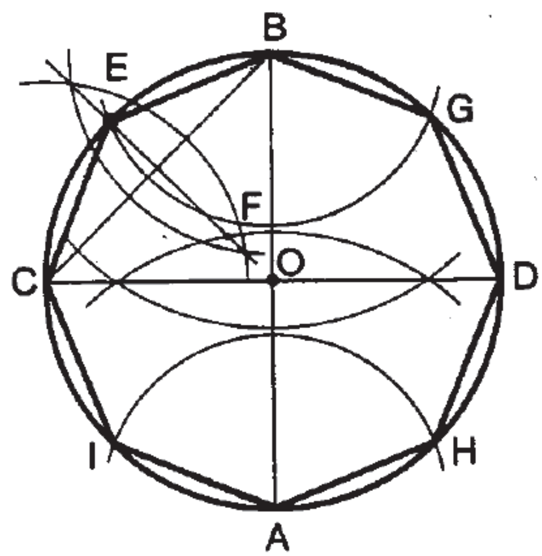
INSCRIBE A REGULAR HEXAGON

1. In the given circle O, draw a diameter $\overline{AB}$.
2. Using A and B as centers and AO as a radius, draw arcs intersecting the circle at C, D, E, and F.
3. Connect A, E, F, B, C, and D to form the hexagon.



INSCRIBE A SQUARE

1. In the given circle O, draw a diameter $\overline{AB}$.
2. Construct another diameter, $\overline{CD}$, which is the perpendicular bisector of $\overline{AB}$.
3. Connect A, D, B, and C to form the square.



INSCRIBE A REGULAR OCTAGON

1. In the given circle O, locate points A, B, C, and D as in the construction for inscribing a square.
2. Draw $\overline{BC}$ and construct $\overline{EF}$, the perpendicular bisector of $\overline{BC}$.
3. Using A and B as centers and BE as a radius, draw arcs intersecting the circle at G, H, and I.
4. Connect A, I, C, E, B, G, D, and H to form the octagon.